Math 250A

Professor Kenneth A. Ribet

Midterm Exam October 9, 1992

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1 (6 points)

Let

$$0 \to A \xrightarrow{\iota} B \xrightarrow{\pi} C \to 0$$

be an exact sequence of abelian groups.

- (4 pts.) a. Let $\rho\colon B \to A$ be a homomorphism such that $\rho \circ \iota = \mathrm{id}_A$. Show that there is a homomorphism $\sigma\colon C \to B$ such that $\pi \circ \sigma = \mathrm{id}_C$.
- (2 pts.) b. Show by example that a homomorphism ρ as in (a) need not exist.

2 (6 points)

Let F be a (covariant) functor from the category of abelian groups to the category of sets.

- (3 pts.) a. Explain in down-to-earth terms what it means for F to be *representable*.
- (3 pts.) b. Suppose that F is the "forgetful" functor which associates to an abelian group A the set A. Is F representable?

3 (7 points)

Prove that all subgroups of $\mathbb{Z} \oplus \mathbb{Z}$ are free abelian groups.

4 (16 points)

Let G be a group of order $13^2 \cdot 7 = 1183$.

- (3 pts.) a. Show that G has a unique subgroup N of order 169.
- (5 pts.) b. Describe $\mathrm{Aut}\, N$ in case N is cyclic, and also in case N is not cyclic.
- (5 pts.) c. Prove that G is abelian if N is cyclic, and solvable in any case.
- (3 pts.) d. Are all groups of order 1183 abelian?