Math 115   Professor Kenneth A. Ribet
Homework assignment #12
Due November 20, 2006

These problems were proposed by Jared Weinstein, who lectured on November 13. They are based on the technique of Hensel's Lemma, which allows one to study congruences mod p^n in terms of congruences mod p.

1. Find all solutions to $x^2 + 3x - 1 \equiv 0 \bmod 3^4$.

2. How many solutions are there to $x^2 \equiv -1 \bmod 25 \cdot 26$?

3. Let p be a prime, and let n be a positive integer. Find the number of solutions to the congruence $x^p \equiv x \bmod p^n$.

4. Let f be a polynomial with integer coefficients and let $f^{(k)}$ be the kth derivative of f. Show that the rational number $f^{(k)}(n)/k!$ is in fact an integer for all integers n.