Yet More Quizzes

May 11, 2013

Math 55 — Ken Ribet

Kim Laine: "All of my quizzes are on my website."

1. Use induction to show that the sum of the squares of the first n positive integers is $\frac{n(n+1)(2n+1)}{6}$.

2. In how many ways can the letters in the word "MISSISSIPPI" be re-arranged if we dont count movement of indistinguishable characters? (You may leave factorials in your final answer.)

3. Find the number of solutions in non-negative integers to the equation $x + y + z + w = 12$.

4. Find a recurrence relation for the number strings of letters from among $\{a, b, c, d\}$ of length n that have 2 consecutive a's.

5. Solve: $a_0 = 0$, $a_1 = 1$, $a_n = a_{n-1} + 6a_{n-2}$.

6. What sequence has $\frac{1}{1+2x}$ as a generating function?

7. Let $A = \{1, 2, 3\}$. Let $R = \{(1, 2), (3, 1), (2, 2), (2, 3)\}$. Give both the matrix representation of R and the digraph for R.

8. Let $A = \{1, 2, 3\}$. Recall that $\Delta = \{(1, 1), (2, 2), (3, 3)\}$ is the diagonal relation. Give an example of binary relations R and S on A such that $R \circ S = \Delta$, yet $R, S \neq \Delta$.

9. Let E_1 and E_2 be two equivalence relations on a set A. Must $E_1 \cup E_2$ be an equivalence relation on A too? (Either prove it must be an equivalence relation or give an example showing it doesn't have to be one.)

10. See Rosen, problem 8.4.23; the GSI changed some of the bounds and added another variable x_4.

11. See Rosen problem 8.2.4(g).

12. What are the three axioms of an equivalence relation?

13. Give an example of: a) a uniformly distributed random variable and b) a binomially distributed random variable.

14. Suppose X and Y are random variables such that $E[XY] = E[X]E[Y]$. Show that $V[X+Y] = V[X] + V[Y]$.

15. Find a recurrence relation and initial conditions for the number of ternary strings containing neither `012` or `210`.

16. Let R be the relation $\{(1,2),(1,3),(4,4)\}$ on the set $\{1,2,3,4\}$. Let P be the property that every vertex has total degree (in-degree minus out-degree) zero. Show that the closure of R with respect to P does not exist.

17. Are the following graphs bipartite? [The question included two pictures. It was pretty easy to see that one of the graphs was bipartite and that the other one wasn't.]

18. Let G be a simple graph. Define a binary relation R on the vertices of G by saying that $(x,y) \in R$ iff there is a finite sequence of $n \geq 1$ vertices $(v_1, \ldots, v_n)$ such that $v_1 = x$, $v_n = y$, and for each i with $1 \leq i < n$, v_i and v_{i+1} are adjacent. Prove that R is an equivalence relation.

19. Consider the graph H which has $\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$ as an adjacency matrix. a) Does H have an Euler path? b) Does H have an Euler circuit?

20. Suppose that G is a connected planar graph that has exactly two regions R_1 and R_2. What are the possible values of $(\deg(R_1), \deg(R_2))$? I.e., what are the possible pairs of degrees of the regions?