Math 55 midterm exam, February 21, 2019

Problem	1	2	3	4	5	6	Total
Points	6	6	5	6	5	6	34

1a. *If S is a subset of a countable set, is S necessarily countable? Explain your answer carefully, outlining a proof or giving a counterexample.*

b. *Suppose that $f : T \to \{1, 2, 3, \ldots\}$ is an onto function. Is the set T necessarily countable?*

2. *Using mathematical and logical operators, predicates, and quantifiers (where the domain consists of all integers) express: "The difference of two positive integers is not necessarily positive."*

3. *Prove or disprove: if A and B are sets, then $\mathcal{P}(A \times B) = \mathcal{P}(A) \times \mathcal{P}(B)$.*

4. *Use the Euclidean algorithm to find the gcd of 39 and 57 and to write the gcd as a linear combination of 39 and 57.*

5. *Find the smallest non-negative integer satisfying the three congruences*

$$x \equiv \begin{cases} -3 & \textit{mod } 19 \\ -3 & \textit{mod } 20 \\ -3 & \textit{mod } 21. \end{cases}$$

(Explain carefully how you got your result.)

6. *Use Bézout's theorem to prove that if a is relatively prime both to b and to c, then a is relatively prime to bc. In symbols:*

$$\gcd(a, b) = \gcd(a, c) = 1 \quad \stackrel{?}{\longrightarrow} \quad \gcd(a, bc) = 1.$$