Math 54	Professor K. A. Ribet
Midterm Exam	September 28, 2005

This exam was an 50-minute exam. It began at 2:10PM. There were 3 problems, for which the point counts were 6, 16 and 8. The maximum possible score was 30. I anticipate that this will be an easy exam. (I'm writing these words three hours before the test.)

> *Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. You may refer to a single 2-sided sheet of notes. Your paper is your ambassador when it is graded. Correct answers without appropriate supporting work will be regarded with great skepticism. Incorrect answers without appropriate supporting work will receive no partial credit. This exam has six pages. Please write your name on each page. At the conclusion of the exam, please hand in your paper to your GSI.*

1. *Find the inverse of the matrix* $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 3 & 2 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 \end{bmatrix}$.

I presume that everyone will do this by Gaussian elimination. The answer should be

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 3 & -1 & -2 & 2 \end{bmatrix}.$$

2. *Label the following statements as TRUE or FALSE, giving a short justification for your choice. There are six parts to this problem, two per page.*

a. *The product of two elementary 3×3 matrices is never an elementary matrix.*

Clearly false because the identity matrix is an elementary matrix. (It corresponds to the operation of multiplication the first row by the number 1.)

b. *If A is an $n \times n$ matrix and B is a column of length n, the equation $AX = B$ has either infinitely many solutions or exactly one solution.*

False because the equation may have no solutions.

c. *If A and B are rectangular matrices for which AB is an invertible square matrix, then A and B are square matrices and both are invertible.*

No we know that AB can be the identity of the appropriate size and yet A and B might not be square (and therefore won't be invertible). For example, A might be the 1×2

matrix with both entries equal to $1/2$, and B might be the 2×1 matrix with both entries to 1. Then AB is the 1×1 matrix with sole entry equal to 1; it's thus the identity matrix I_1 of size 1!

d. *The null space of* $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 9 & 7 \end{bmatrix}$ *is a subspace of* $\mathbf{R}^3$.

Sure, this sounds right.

e. *The set of* 4×4 *matrices* A *for which* $A \begin{bmatrix} 1 \\ 2 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 5 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ *is a subspace of the space of all* 4×4 *matrices.*

No, it's not stable under addition or scalar multiplication.

f. If $\mathbf{v}_1, \ldots, \mathbf{v}_4$ are linearly independent and $\mathbf{v}_2, \ldots, \mathbf{v}_5$ are linearly independent, then the vectors $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_5$ are linearly independent.

This is clearly false. For example, the first and the fifth vectors might be equal.

3. *Find* b *so that* $(-1, b, 2, 3)$ *is in the span of* $(1, 2, 3, 4)$ *and* $(3, 4, 4, 5)$.

You need to look at the system of equations for x and y: $x(1,2,3,4) + y(3,4,4,5) = (-1, b, 2, 3)$. It might help to think of the three vectors in $\mathbf{R}^4$ as vertical column matrices. When is the system consistent? It is consistent exactly when $b = 0$. For this choice of b, the solution is $x = 2$, $y = -1$.

Find all vectors in $\mathbf{R}^4$ *that are perpendicular to both* $(1, 2, 3, 4)$ *and* $(3, 4, 4, 5)$.

Take a vector (x, y, z, w) in $\mathbf{R}^4$. It's perpendicular to the two vectors exactly when $x + 2y + 3z + 4w =$ and $3x + 4y + 4z + 5w = 0$. This is an easy system to analyze by Gaussian elimination or otherwise. The last two variables z and w are independent variables. (You can set them to be s and t if you want.) Then $x = 2z + 3w$ and $y = -(5z + 7w)/2$.