Math 250A, Fall 2001
Homework Assignment #3
Problems due September 18, 2001

a. Let p be a prime, n a positive integer and t an integer that is prime to p. Prove, without reference to group actions, that $\binom{p^n t}{p^n}$ is prime to p. More generally, for $1 \le s \le n$, prove that $\binom{p^n t}{p^s}$ is divisible by p^{n-s} but not by p^{n-s+1}. [To do this problem, it might possibly be useful to know the following fact: Let N be a positive integer, and let S be the sum of the digits in the base p expansion of N. Then $\frac{N-S}{p-1}$ is the exponent of the highest power of p that divides $N!$. If you use this fact, prove it before discussing the main problem.]

b. Suppose that G is a group of order $p^n t$ and that s is as above. Prove that G has a subgroup of order p^s by using Wielandt's method: Consider the action of G on the set of all subsets of G with p^s elements and then argue as we did in class in the case $s = n$.

c. Let G be a group of order $13^2 \cdot 7 = 1183$. Show that G has a unique subgroup N of order 169. Describe $\operatorname{Aut} N$ in case N is cyclic, and also in case N is not cyclic. Prove that G is abelian if N is cyclic, and solvable in any case. Are all groups of order 1183 abelian?

Problems from Lang, Chapter I: 20, 21, 22, 24, 25, 26, 28, 29, 30 (I will try to do at least a couple of these in class before the assignment is due)