Math 250A, Fall 2001
Homework Assignment #2
Problems due September 11, 2001

**a.** For each $n \geq 1$, show that, up to isomorphism, there are only finitely many groups of order $n$.

**b.** For each $h \geq 1$, show that there are only finitely solutions to the equation

$$1 = \frac{1}{t_1} + \cdots + \frac{1}{t_h}. \tag{$*$}$$

in positive integers $t_i$. This problem can presumably be done by elementary methods that have little or nothing in common with the methods of this course. If you want to search for information about this problem on the Web, try the key words "Egyptian fractions". A solution with $h = 7$ that I found on the Web is

$$1 = 1/2 + 1/3 + 1/15 + 1/22 + 1/35 + 1/63 + 1/99.$$

**c.** Let $G$ be a finite group and let $C_1, \ldots, C_h$ be the distinct conjugacy classes of $G$. For each $i$, let $g_i$ be an element of $C_i$ and let $t_i$ be the order of the centralizer of $g_i$ (the group of elements of $G$ that commute with $g_i$). Show that $t_i$ does not depend on the choice of $g_i$ in $C_i$. Establish the relation (*) of the previous problem.

**d.** Let $h$ be a positive integer. Show that, up to isomorphism, there are only finitely many finite groups with exactly $h$ conjugacy classes.

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Problems from Lang, Chapter I: 9 [whoops—this was assigned previously, so forget it], 10, 12, 13, 15, 16, 17, 19. As a number of people have pointed out to me, the displayed formula for part (c) of problem 12 appears to be wrong. I think that the $x$-coordinate of the product should be something like "$x_1 x_2^{h_1}$". In this notation, however, I'm using the construction "$x^h$" as a synonym for "$x^{\psi(h)}$". This means the image of $x$ under the automorphism $\psi(h)$. I personally don't like the exponential notation, because it leads one to write the incorrect formula $x^{hk} = (x^h)^k$. Also, in part (a) of problem 12, the second arrow "$\mapsto$" should be an ordinary right arrow (no vertical bar).