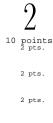
Math 250A

Professor Kenneth A. Ribet Second Midterm Exam November 16, 1992

All modules below are *left* modules.

5 points

Let A be an entire principal ring (principal ideal domain). Explain carefully what it means for an element of A to be an *irreducible* element. Suppose that π is such an element in A. Show directly (without invoking unique factorization) that the ideal (π) is maximal.



Give examples of each of the following, including justification as appropriate: a. A torsion-free Z-module that is not free.

b. A factorial entire ring (unique factorization domain) that is not principal.

- c. A Dedekind ring A that is not factorial. [Don't worry about proving that A is a Dedekind ring: just give some example.]
- d. A ring A such that all A-modules are projective.
- e. A ring A such that all A-modules are injective.

- a. Assume that $M_{\mathfrak{p}}$ is non-zero. Prove that $\operatorname{Ann}(M) \subseteq \mathfrak{p}$.
- b. Suppose that $\operatorname{Ann}(M) \subseteq \mathfrak{p}$ and that M is finitely generated. Prove that $M_{\mathfrak{p}}$ is non-zero.
 - c. Suppose that $\operatorname{Ann}(M) \subseteq \mathfrak{p}$ but that M is not necessarily finitely generated. Is it always true that $M_{\mathfrak{p}}$ is non-zero?

4 3 points

2 pts.

Let A be a commutative entire ring (integral domain). Let \mathcal{F} be a covariant functor from the category of A-modules to the category of sets. Explain precisely what is meant by the statement that \mathcal{F} is representable.

5 3 points

Let X be the abelian group $\{\zeta \in \mathbb{C}^* | \zeta^{p^n} = 1 \text{ for some } n \ge 1\}$. Calculate the ring R = End(X). [First guess what R must be, and then try to prove that your answer is correct.]