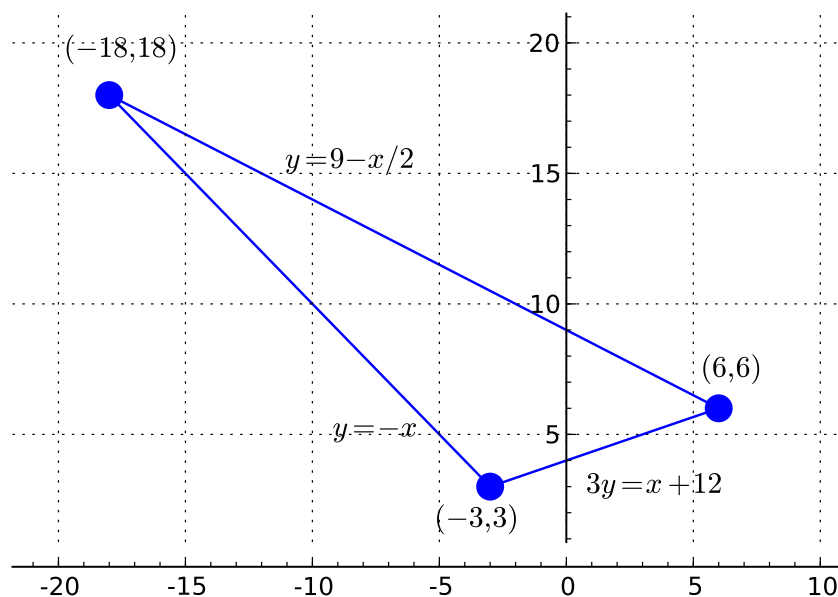


Each problem is worth 10 points. You may consult a single sheet of notes and use a simple electronic calculator while taking this exam. Write carefully and clearly in complete English sentences; be sure to explain your reasoning at each step of your arguments.

1. Find the maximum and minimum values (or explain why there are no such values) for the function $-2x + 3y$ on the region of the plane defined by the inequalities

$$\begin{cases} -x \leq y \leq 9 - x/2 \\ 3y \geq x + 12. \end{cases}$$

Unless I messed up, the indicated region of the plane is the compact region bounded by the triangle in the diagram below:



The vertices of this triangle are the three points $(-18, 18)$, $(-3, 3)$ and $(6, 6)$. By the theorem that we studied (“linear programming”), the extreme values of $-2x + 3y$ occur at the *corners* of the triangle, i.e., at one or more of these three points. The minimum value (which is 6) is at the third point, while the maximum value (which is 90) is at the first point.

2. Describe the set $\{(x, y) \mid 6x^2 + 4y^2 + 48x - 16y + 111 = 0\}$. Is it empty? Is it a conic section? Does it have foci, a directrix, major or minor axes, ...? If so, where in the plane are these objects located? Supply all relevant information; you can even include a crude sketch of the set if you think it will be helpful.

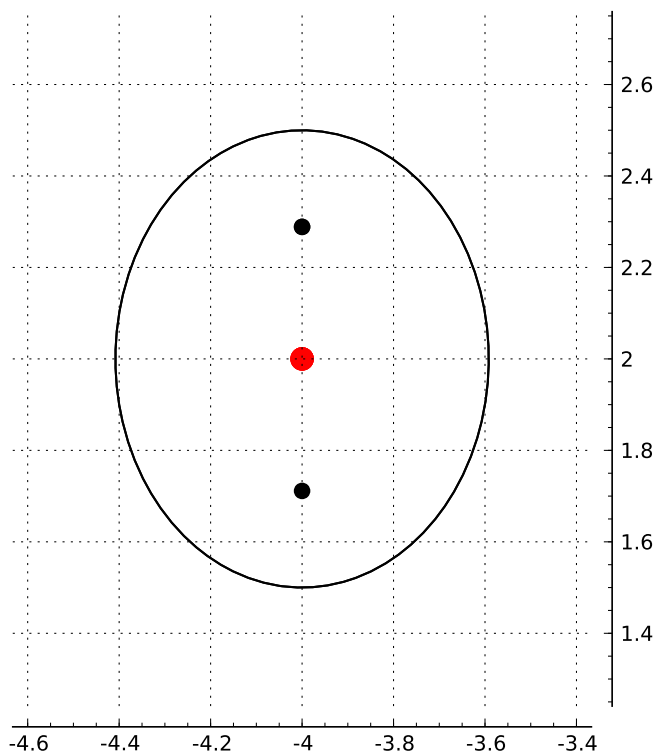
After completing the square (twice), you are likely to end up with

$$6(x + 4)^2 + 4(y - 2)^2 = 1$$

as the equation that's being graphed. This is the ellipse

$$\frac{x^2}{a^2 - c^2} + \frac{y^2}{a^2} = 1$$

with $a = 1/2$ and $c^2 = 1/12$, i.e., $c = \frac{1}{2\sqrt{3}}$. The center of mass, so to speak, is the point $(-4, 2)$, and the foci are aligned vertically. You can fill in the rest....



3. If (a, b) is a point in the plane, its distance to the line $y = -x$ is $\frac{|a+b|}{\sqrt{2}}$. Use this formula to find an equation for the set of points in the plane that are equidistant from the line $y = -x$ and the point $(1, 1)$. What type of figure is described by this equation?

We see from the description that we're talking about the parabola with focus $(1, 1)$ and directrix the line $y = -x$. The equation that I get for the parabola is

$$x^2 + y^2 - 2xy - 4x - 4y + 4 = 0.$$

Notice that this equation looks almost as if it should describe a circle; the "wrinkle" is the presence of the cross term $-2xy$.

