Let $p$ be an odd prime, and let $a$ be a non-zero square in $\mathbb{Z}/p\mathbb{Z}$. The number of $t \in \mathbb{Z}/p\mathbb{Z}$ such that $t^2 - a = 0$ is then 2. How many $t \in \mathbb{Z}/p\mathbb{Z}$ have the property that $t^2 - a$ is a non-zero square? Exploit Problem 10 of §2.2 to find the answer! Next, by subtraction, find the number of $t \in \mathbb{Z}/p\mathbb{Z}$ such that $t^2 - a$ is a non-square (i.e., is not square).

Finally, evaluate the sum $\sum_{t \in \mathbb{Z}/p\mathbb{Z}} \left( \frac{t^2 - a}{p} \right)$, where $\left( \frac{b}{p} \right)$ is the Legendre (or Kronecker) symbol that is defined as follows:

$$\left( \frac{b}{p} \right) = \begin{cases} 
0 & \text{if } b = 0 \mod p, \\
+1 & \text{if } b \text{ is a (non-zero) square } \mod p, \\
-1 & \text{if } b \text{ is a non-square } \mod p.
\end{cases}$$

You can check your answer with sage by typing “sum([kronecker(t^2-1,p) for t in (0,..,p-1)])” for several values of $p$.

Problems from the Book:

§2.4, problems 14a, 14b, 15, 16

§2.5, problems 1, 4

§2.6, problem 3