Math 115 Fall, 2011

Professor K. A. Ribet

Assignment due October 6, 2011

Let p be an odd prime, and let a be a non-zero square in $\mathbf{Z}/p\mathbf{Z}$. The number of $t \in \mathbf{Z}/p\mathbf{Z}$ such that $t^2 - a = 0$ is then 2. How many $t \in \mathbf{Z}/p\mathbf{Z}$ have the property that $t^2 - a$ is a non-zero square? Exploit Problem 10 of §2.2 to find the answer! Next, by subtraction, find the number of $t \in \mathbf{Z}/p\mathbf{Z}$ such that $t^2 - a$ is a non-square (i.e., is not square).

Finally, evaluate the sum $\displaystyle\sum_{t \in \mathbf{Z}/p\mathbf{Z}} \left(\frac{t^2 - a}{p}\right)$, where $\left(\frac{b}{p}\right)$ is the Legendre (or Kronecker) symbol that is defined as follows:

$$\left(\frac{b}{p}\right) = \begin{cases} 0 & \text{if } b = 0 \bmod p, \\ +1 & \text{if } b \text{ is a (non-zero) square } \bmod p, \\ -1 & \text{if } b \text{ is a non-square } \bmod p. \end{cases}$$

You can check your answer with sage by typing "`sum([kronecker(t^2-1,p) for t in (0,..,p-1)])`" for several values of p.

Problems from the Book:

§2.4, problems 14a, 14b, 15, 16

§2.5, problems 1, 4

§2.6, problem 3