Math 115 Fall, 2011

Professor K. A. Ribet

Assignment due November 22, 2011

1. Suppose that $\langle a_0, a_1, \ldots, a_n\rangle$ is the simple continued fraction representation of a rational number. (Recall that our conventions dictate that a_n be at least 2.) Define the numbers h_n and k_n as usual. Establish the formula

$$\frac{k_n}{k_{n-1}} = \langle a_n, a_{n-1}, \ldots, a_2, a_1\rangle.$$

(Hint: it seems helpful to recall the recursive formula that defines k_i in terms of a_i and previous ks.)

2. Suppose that p is an odd prime number and that u is the square root of -1 mod p that satisfies $1 \le u \le (p-1)/2$. Take u/p to be the rational number of part (1). In other words, write

$$\frac{u}{p} = \langle a_0, a_1, \ldots, a_n\rangle.$$

Show that $k_n = p$ and that $h_n = u$. Using the formula $h_n k_{n-1} - k_n h_{n-1} = (-1)^{n-1}$, show that n is even and that $k_{n-1} = u$.

3. Combining (1) and (2), show that

$$p/u = \langle a_n, a_{n-1}, \ldots, a_2, a_1\rangle.$$

Conclude that the strings $(a_n, a_{n-1}, \ldots, a_2, a_1)$ and $(a_1, \ldots, a_n)$ are identical.