1. Suppose that \( \langle a_0, a_1, \ldots, a_n \rangle \) is the simple continued fraction representation of a rational number. (Recall that our conventions dictate that \( a_n \) be at least 2.) Define the numbers \( h_n \) and \( k_n \) as usual. Establish the formula

\[
\frac{k_n}{k_{n-1}} = \langle a_n, a_{n-1}, \ldots, a_2, a_1 \rangle.
\]

(Hint: it seems helpful to recall the recursive formula that defines \( k_i \) in terms of \( a_i \) and previous \( k \)s.)

2. Suppose that \( p \) is an odd prime number and that \( u \) is the square root of \(-1\) mod \( p \) that satisfies \( 1 \leq u \leq (p - 1)/2 \). Take \( u/p \) to be the rational number of part (1). In other words, write

\[
\frac{u}{p} = \langle a_0, a_1, \ldots, a_n \rangle.
\]

Show that \( k_n = p \) and that \( h_n = u \). Using the formula \( h_n k_{n-1} - k_n h_{n-1} = (-1)^{n-1} \), show that \( n \) is even and that \( k_{n-1} = u \).

3. Combining (1) and (2), show that

\[
p/u = \langle a_n, a_{n-1}, \ldots, a_2, a_1 \rangle.
\]

Conclude that the strings \( (a_n, a_{n-1}, \ldots, a_2, a_1) \) and \( (a_1, \ldots, a_n) \) are identical.