Math 114

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Midterm Exam   April 6, 1992

1

15 points

Cite examples of each of the following:

    a. A normal extension of fields which is not separable.

    b. A separable extension of fields which is not normal.

    c. A K-monomorphism $L \to L$ which is not an automorphism.

    d. Two automorphisms of a field which do not commute with each other.

    e. A Galois extension of degree 114.

2

18 points

Let p be the prime number 101. Let $L = \mathbb{Q}(e^{\frac{2\pi i}{p}})$.

    a. Find $[L : \mathbb{Q}]$.

    b. Is the extension $L/\mathbb{Q}$ separable?

    c. Is the extension $L/\mathbb{Q}$ normal?

    d. Describe the group of automorphisms of the field L.

    e. How many subfields of L have degree 25 over $\mathbb{Q}$?

3

12 points

Let L/K be a finite Galois extension, with Galois group G. For $x \in L$, set $\mathcal{N} x := \prod_{\sigma \in G} \sigma x$.

    a. Show $\mathcal{N} x \in K$.

    b. If L is a finite field, show that $\mathcal{N}$ is the map $x \mapsto x^i$, where $i = (L^* : K^*)$.

    c*. If L is a finite field, show that $\mathcal{N}$ maps L *onto* K.