Math 114

Professor Kenneth Alan Ribet

Midterm Exam February 24, 1992

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1 (4 points) Which integers d satisfy

$$d\mathbb{Z} = \{\, 51n - 68m \mid n, m \in \mathbb{Z} \,\}?$$

2 (20 points) Cite concrete examples of:

a. An irreducibility criterion named for a German mathematician;
b. An infinite field of characteristic 5;
c. An algebraic extension of fields that has infinite degree;
d. A reducible polynomial with no roots;
e. An irreducible polynomial of degree 96;
f. An irreducible cubic polynomial over $\mathbb{F}_2$;
g. A French mathematician who was killed in a duel.
h. A non-zero polynomial over some field K which vanishes on all elements of K.

3 (12 points) Explain why each cannot exist:

a. A subfield with exactly 4 elements of a field with exactly 8 elements.
b. A non-trivial factorization of $3t^4 - 25t^3 + 5t^2 - 10$ over $\mathbb{Q}$.
c. An isomorphism of rings $2\mathbb{Z} \approx 3\mathbb{Z}$.
d. An irrational number which may be written both as $f(\sqrt[3]{2})$ and as $g(\sqrt{3})$, for some f and g in $\mathbb{Q}[X]$.

4 (9 points) Let L be a field and let K be a subfield of L. Suppose that α is an element of L, and let $R = K[\alpha]$ be the ring of polynomial expressions in α, with coefficients in K. Consider the group $U = R^*$ of units of R: the set of elements of R with inverses in R. Show that

$$U = \begin{cases} K \setminus \{0\} & \text{if } \alpha \text{ is transcendental over } K; \\ R \setminus \{0\} & \text{if } \alpha \text{ is algebraic over } K. \end{cases}$$

There are a total of 45 points. Solutions will be distributed at the end of the hour.