Math 113
Professor Ken Ribet
Homework due October 31, 2013

1. Let n be an integer ≥ 3 and let G be the dihedral group D_{2n}. Find the number of 2-Sylow subgroups of G and describe them as well as you can.

2. How large is the conjugacy class of $(1\,2)(3\,4)(5\,6\,7)(8\,9\,10)$ in S_{10}? How large is the stabilizer of this permutation in S_{10}? Describe the stabilizer as well as possible.

3. Same as question #2, but with S_{10} replaced by A_{10}.

4. Write out carefully a proof of the following theorem: *Suppose that G is a finite group and that p is a prime number dividing the order of G. Suppose that P is a p-Sylow subgroup of G and that H is a subgroup of G whose order is divisible by p. Then there is an element g of G so that the intersection $(gPg^{-1}) \cap H$ is a p-Sylow subgroup of H.* Be sure to write in complete English sentences; pay attention to spelling and grammar.

5. Find as many different groups as you can that have exactly three conjugacy classes. "Different" means not isomorphic to each other.

6. Show that every finite group with more than two elements has an automorphism other than the identity.