Math 113H

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First Midterm Exam February 18, 1991

1. Let G be a cyclic group of order 6^{100}. How many subgroups does G have? (Don't just write down a number; give some explanation.)

2. Let G be a group of order 11^2 which is not cyclic. How many elements of order 11 are there in G? How many subgroups does G have?

3. Find an integer x such that $x \equiv 23 \bmod 69$ and $x \equiv 34 \bmod 397$. (Don't bother simplifying your answer if it is complicated.) The identity $4 \cdot 397 = 23 \cdot 69 + 1$ will probably be useful.

4. Here are two relations on the set of rational numbers. Which are equivalence relations? (Explain your answers.)
a. Two rational numbers are related if their sum may be written $\frac{p}{q}$ with p and q integers such that p is even and q is odd.
b. Two rational numbers are related if their difference may be written $\frac{p}{q}$ where p and q are integers such that q is not divisible by 4.

5. Let g be an element of a finite group G. Define a map $\phi\colon \mathbf{Z} \to G$ by $\phi(n) = g^n$ for $n \in \mathbf{Z}$. Show that ϕ is a homomorphism of groups. Prove that the kernel of ϕ consists precisely of the multiples of the order of g.