Can you please post solutions to problem 7 part d of 4.1 and also to problems 11 and 12 of 4.2?

**Problem 7d of §4.1:** Suppose for each $a \in A$ that $G_a$ is a maximal subgroup. Let $B$ be a block. Since $B$ is non-empty, there is an element $a$ of $B$. We are told to use part (a), so let's do that: we find that $G_B$ contains $G_a$, so $G_B = G$ or $G_B = G_a$. If $G_B = G$, then $\sigma(B) = B$ for all $\sigma \in G$. Since the various $\sigma(B)$ cover $G$ (part (b)), we have $B = G$. If $G_B = G_a$, we want to prove that $B = \{a\}$. Take $b \in B$. Because $G$ is transitive, there is a $\sigma \in G$ with $\sigma(a) = b$. The intersection $\sigma(B) \cap B$ is non-empty: it contains $b$. Hence $\sigma(B) = B$ by the definition of "block." It follows that $\sigma$ lies in $G_B = G_a$, so that $\sigma$ fixes $a$. Thus $b = a$; this gives that $B = \{a\}$, since $b$ was an arbitrary element of $B$.

Suppose now that $G$ is a transitive group acting on $A$ for which one of the $G_a$ is non-maximal. Think of $A$ as the orbit of $a$ (OK because of transitivity) and write $O(a)$ as $G/G_a$. Then $A = G/G_a$. Take a subgroup $H$ of $G$ that's strictly between $G_a$ and $G$; this exists by hypothesis. Consider the subset $B = H/G_a$ of $A = G/G_a$. This subset has more than one element and doesn't fill out all of $A$; thus it's a non-trivial block if it's a block. To show it's a block we have to show that $g(B) = B$ whenever $g(B)$ and $B$ have an element in common. Well, $B$ and $g(B)$ correspond to subsets of $G/G_a$; $B$ is the set of all $hG_a$ with $h \in H$ and $g(B)$ is the set of all $ghG_a$ with $h \in H$. Suppose that these subsets have an element in common; then there are $h, h' \in H$ such that $hG_a = gh'G_a$. This gives $h^{-1}gh' \in G_a$. Now if $x = h^{-1}gh'$, then $x$ is in $G_a$, which is contained in $H$, so $x \in H$. It follows that $g$ lies in $H$, so $g(B) = B$.

**§4.2, problem 11:** When we compute the cycle structure of a permutation $\sigma \in S_n$, we are figuring out the different orbits of elements in $\{1, 2, \ldots, n\}$ under the cyclic group generated by $\sigma$. Two letters in $\{1, 2, \ldots, n\}$ belong to the same cycle if and only if you can get from one to the other by some power of $\sigma$. In this exercise, $S_n$ is replaced by the permutation group on the set underlying the group $G$ and we are taking $\sigma$ to be the permutation induced by multiplication by $x$. The orbits that we are talking about are of the form $t, xt, x^2t, \ldots, x^{n-1}t$ where $t$ is an element of $G$; here $n$ is the order of $x$ (as in the statement of the problem). The cycle decomposition of $\sigma = \pi(x)$ thus consists of a bunch of cycles of length $n$. The number of cycles is determined by the fact that $G$ is covered completely by disjoint orbits: if $|G| = nm$, there are $m$ different cycles. The sign of $\pi(x)$ is then $\epsilon^m$, where $\epsilon$ is the sign of a cycle of length $n$. We know that cycles of length $n$ have sign $(-1)^{n-1}$, so the sign of $\pi(x)$ is $\left((-1)^{n-1}\right)^m$. This sign is $-1$ precisely when $n$ is even and $m$ is odd.

**§4.2, problem 12:** If $\pi(G)$ contains an odd permutation, then the homomorphism $\operatorname{sgn} \circ \pi : G \to \{\pm 1\}$ is surjective. (I write "sgn" for the sign of a permutation.) The kernel of this homomorphism has index 2 in $G$.