Homework assignment #10, due October 31

• §5.4: 13, 17, 18, 20, 23 (can use quotient spaces here if you want), 24, 25, 41
• §6.1: 19, 20

• Summarize and complete the discussion about the Iwasawa decomposition that took place at the end of Friday’s class (i.e., the class on October 24). Namely, for $n \geq 1$, show that every $n \times n$ invertible matrix with coefficients in $F$ can be written as a product $N \cdot D \cdot U$ where $N$ is upper-triangular and has 1’s on the diagonal, $D$ is a diagonal matrix whose diagonal entries are positive real numbers, and $U$ is a unitary matrix. Recall that $F$ is either $\mathbb{R}$ or $\mathbb{C}$ and that $U$ is unitary if its conjugate transpose is its inverse. Real unitary matrices are called orthogonal.

• Suppose that $F = \mathbb{R}$ and that $A$ is an $n \times n$ matrix over $F$ that satisfies $\|Ax\| = \|x\|$ for all $x \in F^n$. Show that $A$ is unitary (i.e., orthogonal). In the case $F = \mathbb{C}$, can we draw the analogous conclusion? In other words, if $\|Ax\| = \|x\|$ for all $x \in \mathbb{C}^n$, is $A$ a unitary matrix?