Math H110 Professor K. Ribet

Homework assignment #10, due October 31

- §5.4: 13, 17, 18, 20, 23 (can use quotient spaces here if you want), 24, 25, 41

- §6.1: 19, 20

- Summarize and complete the discussion about the Iwasawa decomposition that took place at the end of Friday's class (i.e., the class on October 24). Namely, for $n \geq 1$, show that every $n \times n$ invertible matrix with coefficients in F can be written as a product $N \cdot D \cdot U$ where N is upper-triangular and has 1's on the diagonal, D is a diagonal matrix whose diagonal entries are positive real numbers, and U is a unitary matrix. Recall that F is either $\mathbf{R}$ or $\mathbf{C}$ and that U is unitary if its conjugate transpose is its inverse. Real unitary matrices are called orthogonal.

- Suppose that $F = \mathbf{R}$ and that A is an $n \times n$ matrix over F that satisfies $\|Ax\| = \|x\|$ for all $x \in F^n$. Show that A is unitary (i.e., orthogonal). In the case $F = \mathbf{C}$, can we draw the analogous conclusion? In other words, if $\|Ax\| = \|x\|$ for all $x \in \mathbf{C}^n$, is A a unitary matrix?