Suppose that $p = 0$ in F. Then we'd like to find $p \times p$ matrices A and B such that $AB - BA = I$, where I is the identity matrix of size p. Equivalently, we'd like to exhibit a p-dimensional vector space V together with linear maps T and U from V to V such that $UT - TU = I$, where I now is the identity map of V.

Here is an intrinsic version of the solution that was proposed today in class by Boris (in the front row). Let F be a field in which p is 0 and let $W = F[x]$ be the space of all polynomials (of all degrees) over F in the variable x. Then x^pW is the subspace of polynomials that have no terms of degree $< p$. Let $V = W/x^pW$. Then V is basically the space of polynomials of degree $< p$, except that we agree to view polynomials of arbitrary degree as elements of V by tossing away all terms involving x^p, x^{p+1}, and so on.

Let $T : V \to V$ be the linear map "multiplication by x." Then $T(1) = x$, $T(x) = x^2$, and so on; note that $T(x^{p-1}) = x^p = 0$. Let U be the map "differentiation with respect to x." This map is really well defined on V because the derivative of any polynomial in x^pW is again in x^pW.

Consider $UT - TU$. This takes 1 to the derivative of x, which is 1. It takes x to $2x - x = x$. It takes x^2 to $3x^2 - 2x^2 = x^2$. And so on. At the end of the string of basis vectors, it takes x^{p-1} to $0 - (p-1)x^{p-1} = x^{p-1}$. Hence $UT - TU$ is the identity map on V.