Let $T$ be a linear transformation on a vector space $V$ over $F$. Suppose that $v_1, v_2, \ldots, v_k \in V$ are eigenvectors of $T$ that correspond to distinct eigenvalues. Assume that $W$ is a $T$-invariant subspace of $V$ that contains the vector $v_1 + v_2 + \cdots + v_k$. Show that $W$ contains each of $v_1, v_2, \ldots, v_k$. [It may help to argue by induction on $k$.]