Let V and W be vector spaces over F. Let $T : V \to W$ be a linear transformation. Show that T is one-to-one if and only if T carries linearly independent subsets of V to linearly independent subsets of W. [Caution: linearly independent sets can have an infinite number of elements.]