Math 10B

PROFESSOR KENNETH A. RIBET

First Midterm Examination

February 16, 2016

2:10–3:30 PM, Andersen Auditorium

Please write your NAME and SID clearly:

GSI Name:

Please put away all books, calculators, cell phones and other devices. You may consult a single two-sided sheet of notes. Please write carefully and clearly, *using words* (not just symbols). Remember that the paper you hand in will be your only representative when your work is graded.

Please ensure that your name has been written clearly on each page of your exam. Your paper will be scanned and will be processed using Gradescope. It is essential that you hand in the ten pages that you have received (including this cover sheet) and that the order of the pages be preserved.

At the conclusion of the exam, hand your paper in to your GSI. (For the handing-in, Ken Ribet will play the role of Liz Ferme.)

It's OK to leave answers in terms of numbers like $C(n,k)$, $n!$, $P(n,k)$ but *not* OK to leave answers in terms of Stirling numbers. Also, don't worry about evaluating or simplfying arithmetic expressions like $3 \times 4 - 7$; you can do this if you want, though!

There are no induction problems on this midterm.

Each of the seven problems is worth five points. The maximum possible score on this midterm is $7 \cdot 5 = 35$.

Math 10B first midterm, February 16, 2016

NAME: _____ SID: _____

1. A child rolls a fair die six times in succession. What is the probability that she rolled each of the numbers 1–6 exactly once?

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2. Calculate the number of ways of rearranging the letters `CALBEARS` so that `E` appears somewhere to the left of `S`.

NAME: _____ SID: _____

3. How many ways are there to put 12 indistinguishable burritos and 12 indistinguishable tacos into six distinguishable boxes?

Math 10B first midterm, February 16, 2016

NAME: _____ SID: _____

4. In how many ways can three students be assigned to six different discussion sections if each discussion section can receive at most one student?

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5. In how many ways can six students be assigned to three different discussion sections, if each discussion section must receive at least one student?

NAME: _____ SID: _____

6. For $n \geq 0$, let $F(n)$ be the number of bit strings of length n that do not contain `00`. We have $F(0) = 1$, $F(1) = 2$, $F(2) = 3$,....

a. For $n \geq 2$, explain why the number of bit strings of length n that start with `0` and do not contain `00` is equal to $F(n-2)$.

b. For $n \geq 1$, explain why the number of bit strings of length n that start with `1` and do not contain `00` is equal to $F(n-1)$.

c. Calculate the number of bit strings of length 4 that do not contain `00`, using the insights of parts **a** and **b** (or otherwise, if you prefer).

NAME: _____ SID: _____

7. Before going on vacation for a week, you ask your spacey friend to water your ailing plant. Without water, the plant has a 90 percent chance of dying. With water, it has a 20 percent chance of dying. The probability that your friend will forget to water it is 30 percent.

a. What is the probability that your plant will be dead at the end of the week?

b. If the plant is dead when you return, what is the probability that your friend forgot to water it?

Math 10B first midterm, February 16, 2016

NAME: _____ SID: _____

(Scratch Paper, Page 1)

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(Scratch Paper, Page 2)

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