UC Berkeley Math 10B, Spring 2015: Homework 9

Due Wednesday, April 13

Separable Nonlinear ODEs

1. Determine if the ODE below are separable; if so, set up (but do not solve) the resulting integrals.

   (a) $y' = \dfrac{1}{yt}$

   (b) $y' = \sin(y)$

   (c) $y' = \ln(y^t) + t$

   (d) $y' = t + y$

   (e) $y' = e^{t+y}$

2. Solve the differential equation $y' = \dfrac{t^2}{y}$

3. Solve $y' + y^2 \sin t = 0$

4. Solve $y' = \dfrac{t^2}{y(1+t^2)}$

5. Solve the initial value problem

$$\begin{cases} y' = \dfrac{(1+3t^2)}{3y^2 - 6y} \\ y(0) = 1 \end{cases}$$

   It is OK to leave the answer in implicit form.

6. Solve

$$ty' = \sqrt{1 - y^2}$$

Second-order linear ODEs

7. Find the general solution to the following second-order ODEs:

   (a) $y'' + 2y' - 3y = 0$

   (b) $6y'' - y' - y = 0$

   (c) $y'' + 5y' = 0$

   (d) $y'' + 2y' + 3y = 0$

8. Find a second-order linear ODE whose general solution is $y = C_1 e^{2t} + C_2 e^{-3t}$.

9. Find the solution of the initial value problem

$$\begin{cases} 2y'' - 3y' + y = 0 \\ y(0) = 2, y'(0) = \dfrac{1}{2}. \end{cases}$$

10. Solve the initial value problem

$$\begin{cases} y'' + 2ay' + (a^2+1)y = 0 \\ y(0) = 1,\ y'(0) = 0 \end{cases}$$

11. Solve the initial value problem

$$\begin{cases} y'' - y' - 2y = 0 \\ y(0) = \alpha, y'(0) = 2. \end{cases}$$

Find α so that the solution approaches zero as $t \to \infty$

12. Consider the initial value problem

$$\begin{cases} y'' + 2y' + 6y = 0 \\ y(0) = 2,\ y'(0) = \alpha. \end{cases}$$

Here α is a real constant.

(a) Find the solution y of this problem.

(b) Find α so that $y = 0$ when $t = 1$.

13. Find the general solution of the ODE

$$t^2y'' + 3ty' - 3y = 0.$$

(*Hint*: This linear, second-order ODE has variable coefficients and so the methods from class will not work. Instead, look for solutions of the form $y = t^r$.)

Complex numbers

These problems are *not* to be handed in. They're included to check that you all are OK with the material on "Complex Numbers" in Appendix B of the online "textbook" `Dynamics.pdf`. If these problems look like they're from outer space, try reading the appendix and then come to ask for help if they still seem weird to you.

14. Euler's formula states that, for $i = \sqrt{-1}$, we have

$$e^{ix} \ = \ \cos x \ + \ i \cdot \sin x.$$

Use Euler's formula to write the following expressions in the form $x + iy$:

(a) e^{3+4i}

(b) $e^{7\pi i}$

(c) π^{-i+2}

15. Use Euler's formula to show that, for all real numbers t,

$$\cos t = \frac{(e^{it} + e^{-it})}{2} \quad \text{and} \quad \sin t = \frac{(e^{it} - e^{-it})}{2i}.$$