UC Berkeley Math 10B, Spring 2015: Homework 8

Due: March 30

Partial fractions

1. The polynomial $f(x) = x^3 - 5x^2 + 3x + 7$ has three distinct roots. Find the sum of the three roots, and also find the product of the three roots.

2. Integrate
$$\int \frac{x + 27}{x^2 - 9}\, dx.$$

3. Integrate
$$\int \frac{1}{x^2 - 6x + 9}\, dx.$$

4. Integrate
$$\int \frac{7x - 26}{x^2 - 6x - 16}\, dx.$$

5. Integrate
$$\int \frac{8x - 36}{(x - 5)^2}\, dx.$$

6. Integrate
$$\int \frac{18}{(x + 3)(x^2 - 9)}\, dx.$$

7. Integrate
$$\int \frac{18}{(x + 3)(x^2 + 9)}\, dx.$$

Linear first-order ODE

8. Determine whether the differential equation is a linear, nonhomogeneous first-order ODE:

   (a) $y' + \cos t = y$
   (b) $y' + \cos y = \tan t$
   (c) $yy' + ty = t^2$
   (d) $ty + \sqrt{t} = e^t y'$

9. Solve the differential equation $ty' + y = \sqrt{t}$

10. Solve the differential equation $y' + 2y = 2e^t$.

11. Solve the initial value problem $y' - 2ty = 3t^2 e^{t^2}$, $y(0) = 5$.

12. (Radiocarbon dating) Carbon-14 is an isotope of carbon. Living things contain a fixed percentage of C-14 because they are in equilibrium with the atmosphere; but once they die, they stop taking in new C-14 and the amount present begins to decay. The rate of decay of C-14 is proportional to the quantity of C-14; that is:

$$C' = kC$$

where C is the amount of C-14 present.

(a) The half-life of a substance is the amount of time it takes for half of the original quantity to decay. The half-life of C-14 is 5600 years. Use this fact to determine the value of the rate constant k.

(b) Suppose the equilibrium level of C-14 is C_0. Solve the IVP

$$C' = kC, \quad C(0) = C_0$$

(c) Suppose that corn kernels found in an archaeological site contain 20% of the equilibrium level of C-14. Determine how many years ago the corn was harvested.

13. A tank is used in hydrodynamic experiments. After one experiment the tank contains 200 L of a dye solution with a concentration of $1\,\mathrm{g/L}$. To prepare for the next experiment, the tank is to be rinsed with fresh water flowing in at a rate of $2\,\mathrm{L/min}$, the well-stirred solution flowing out at the same rate.

    Find the time that will elapse before the concentration of the dye in the tank reaches 1% of its original value.

14. Solve the differential equation $(t^2 - 3t + 2) \cdot y' + 7 = 0$.