UC Berkeley Math 10B, Spring 2016: Homework 7

Due: March 16

Recursion equations

1. Determine which of the following are linear homogeneous constant coefficient recursion relations. Find the degree of those that are.

   (a) $a_n = 3a_{n-1} + 2a_{n-4}$
   (b) $a_n = a_{n-1}^2 - 2$
   (c) $a_n = a_{n-2} + 2$
   (d) $a_n = a_{n-1} + 7a_{n-3}a_{n-1}$

2. Find all solutions of
$$a_n = a_{n-1} + 2$$

3. Solve
$$y_n - y_{n-1} = 0.03y_{n-1} - 0.07, \quad y(0) = y_0$$

4. Solve the recurrence
$$a_n = a_{n-1} + 2a_{n-2}, \qquad a_0 = 2, a_1 = 1.$$

5. Determine values of A and B such that $a_n = An + B$ is a solution of the recurrence relation
$$a_n = 2a_{n-1} + n + 5.$$

6. Solve
$$a_n = 2a_{n-1} + 3a_{n-2}, \qquad a_0 = 1, a_1 = 3.$$

Differential equations

1. At 7PM, a large pizza is taken from a 415^oF oven to a 65^oF dining room. At 7:08PM, the pizza has cooled to 135^oF. What is the temperature of the piece which remains at 7:16PM? (Assume the validity of Newton's law of cooling, which was discussed in class on March 10.)

2. A spherical raindrop evaporates at a rate proportional to its surface area. Write a differential equation for the volume $V(t)$ of the raindrop as a function of time.

3. (Compound interest) If one invests an amount A_0 with interest rate r compounded annually, after one year it will be worth $A_0(1+r)$.

   (a) Find an expression for $A(t)$, the value of the investment after t years.

   (b) Suppose that the investment is compounded n times per year. Then, assuming the investment rate during each period is r/n, find an expression for $A(t)$. (Make sure the case $n = 1$ is the same as your answer to part (a)!)

   (c) Suppose that the investment is *continuously compounded*; that is, take the limit as $n \to \infty$. What is the expression for $A(t)$ now? What differential equation does $A(t)$ satisfy?

4. Find a solution to the initial value problem

$$y' = 3y,\ y(0) = 1.$$

   Decide whether your solution is unique.

5. Find a solution to the initial value problem

$$y' = 2y^{1/2},\ y(0) = 0.$$

   Decide whether your solution is unique.