UC Berkeley Math 10B, Spring 2016: Homework 6

Due March 9

Disclaimer: Most of these problems concern material that is discussed in the online "textbook" (and elsewhere, e.g., in Wilipedia) but will *not* be discussed in any detail in the March 3 lecture. For this particular assignment, you're more on your own than you have been. (I will be only marginally helpful on piazza and in office hours.) After this week, we'll move back to topics that I know well.

Sampling and estimation

1. You flip a biased coin 100 times and get heads 63 times. Use this to construct a 95% confidence interval for p, the probability of getting heads.

2. A random variable X is Poisson with unknown parameter λ, the expected number of events per day. You observe over 10 days these numbers of events: $4, 3, 2, 6, 5, 4, 1, 3, 4, 3$.

   Use these data to construct a 95% confidence interval for λ.

3. The number of times a machine needs resetting on a night shift follows a Poisson distribution. On three randomly selected nights it was reset 9, 5 and 11 times. Calculate a 95% confidence interval for the average number of times it needs resetting per night.

Hypothesis testing, chi-squared tests

4. Determine the value of the integral

$$\int_0^\infty x^2 e^{-x/2} dx.$$

   Hint: use your understanding of statistics and the gamma function.

5. Suppose we roll a 6-sided die 60 times and observe the outcomes in the following table.

Value	1	2	3	4	5	6	Total
Observed Frequency	9	13	8	7	15	8	60
Expected Frequency	10	10	10	10	10	10	60

   Compute the χ^2 statistic for this data. Do we have enough evidence to reject the null hypothesis H_0 that the die is fair?

6. Same question as the previous problem, but for the data

Value	1	2	3	4	5	6	Total
Observed Frequency	8	13	9	7	13	10	60
Expected Frequency	10	10	10	10	10	10	60

7. We roll two 6-sided dice 100 times and record the outcomes for the sum of the dice in the following table.

Value	2	3	4	5	6	7	8	9	10	11	12	Total
Observed Frequency	6	10	9	13	13	12	11	10	7	5	4	100

Calculate the expected frequencies, given the null hypothesis H_0 that both dice are fair. Compute the χ^2 statistic for this data. What is the p-value? Do we have enough evidence to reject the null hypothesis?

8. Same problem, but with the data from the following table:

Value	2	3	4	5	6	7	8	9	10	11	12	Total
Observed Frequency	6	10	9	11	13	17	8	10	5	6	5	100

9. This table records the observed frequencies of joint outcomes for a random variable X taking the three values x_1, x_2, x_3 and a random variable Y taking the two values y_1, y_2.

	x_1	x_2	x_3
y_1	827	159	184
y_2	177	239	135

Construct a table showing the expected frequencies under the null hypothesis H_0 that X and Y are independent. Find the χ^2 test statistic and compute its p-value. Do we have enough evidence to reject the null hypothesis?

10. Same problem as the previous one, but now the random variable Y that takes on three values, and these are our observed data:

	x_1	x_2	x_3
y_1	16	29	23
y_2	17	62	34
y_3	77	42	15