UC Berkeley Math 10B, Spring 2016: Homework 5

Due March 2

Discrete distributions

1. Suppose that the number of large earthquakes in California a given year is a Poisson random variable with $\lambda = 2$. What is the probability of 5 earthquakes in a given year?

2. You roll 2 fair dice, and repeat this process until both dice show the same number. What is the probability that it takes you more than 10 rolls of the pair of dice until this happens?

3. A detective gathers information about a bank robbery by interviewing citizens of a town. There were 20 people present at the bank on the day of the robbery, the detective chooses 50 people to interview at random, and she never interviews the same person twice. What is the probability of her finding precisely three witnesses if the population of the town is 300?

4. Suppose we perform n Bernoulli trials, each of which has probability of success p. Compute the probabilities of

   (a) no failures,

   (b) at least one failure,

   (c) two failures,

   (d) at most one failure.

5. Suppose that the number of frogs observed in a pond each day is a Poisson random variable with parameter $\lambda = 0.1$. What is the probability that

   (a) you observe precisely one frog today?

   (b) you observe more than one frog today?

6. Suppose pens are sold in packages of 20. The manufacturer guarantees that no more than one pen per package will be defective, and will replace any package with two or more defective pens. Supposing that each pen is defective with probability 0.02, what is the probability that a package will need to be replaced?

7. The average number of lions seen on a 1-day safari is 5. Assuming a Poisson distribution, what is the probability that tourists will see fewer than four lions in the next 1-day safari?

Expected value and variance

8. Suppose that Y is a Poisson random variable with expected value 25. What is the probability that $Y = 2$?

9. Let X be the number of heads in ten flips of a fair coin. Determine $E[X]$ and $\mathrm{Var}[X]$.

10. Suppose X is a binomial random variable with standard deviation 2 and expected value 8. What is the probability that $X = 3$?

11. A coin is biased so that the probability a head comes up when it is flipped is 0.6. What is the expected number of heads that come up when it is flipped 10 times?

12. What is the expected sum of the numbers that appear on two dice, each biased so that a 3 comes up twice as often as each other number?

13. Suppose that we roll a pair of fair dice until the sum of the numbers on the dice is seven. What is the expected number of times we roll the dice?

14. Let X be the random variable that equals the number of tails minus the number of heads when n fair coins are flipped.

    (a) What is the expected value of X?

    (b) What is the variance of X?

15. You toss a fair coin until you get a head. If it takes you n flips before the first head, you get paid 2^n dollars. Let X denote the number of dollars you win. Show that $E[X] = \infty$. (This is called the St. Petersburg Paradox.)

16. Suppose you repeatedly roll a fair die, and stop once you roll some number twice in a row. What is the expected number of rolls needed?

    (Hint: For each roll except the first, what would you consider a "success"?)

17. Assume Ω is a finite set, $X : \Omega \to \mathbb{R}$ is a random variable with range R, and $f : \mathbb{R} \to \mathbb{R}$ is any function. Show that
$$E[f(X)] = \sum_{x \in R} f(x) P(X = x).$$