UC Berkeley Math 10B, Spring 2016: Homework 4

Due Wednesday, February 24

Bayes' Theorem

1. The rate of migraines in the general population is 10%. Suppose that 5 % of the population is taking a certain drug, and 30% of that fraction suffer from migraines as a side effect. What is the probability that a migraine sufferer is taking the drug?

2. Suppose that the probability of getting pulled over for erratic driving is 60% in Berkeley and 40% in Reno. Assume you are an erratic driver. If you flip a fair coin to decide which city to visit, what is the probability that you will get pulled over?

3. Suppose that 28% of voters classify themselves as Democrats, 31% as Republicans, and 41% as independents. Assuming that the voter turnouts were 45%, 32%, and 26 % for Democrats, Republican and independents respectively, what is the probability that a person who voted is

   (a) a Democrat?

   (b) a Republican?

   (c) an independent?

4. Suppose that a certain drug test at an athletic event is 99% correct for both users and non-users. Suppose that 0.1% of athletes use the illegal drug.

   What is the probability that you do not use drugs, given that the test says you do?

5. A restaurant has three chefs. Suppose that that if a patron eats a meal prepared by Chef A, B, or C, the probability of dissatisfaction is 0.02, 0.03, and 0.05, respectively. Suppose that Chef A makes 50% of the meals, B makes 20%, and C makes 30% of the meals.

   If a meal was a failure, what is the probability that it was prepared by Chef A?

Independence

1. Consider families with two children, are they following two events independent ?
   $A =$ the family has two boys, $B =$ the family has at least one boy.
   (Assume that all four outcomes are equally likely.)

2. Show that if the events A and B are independent, then so are the events A and B^c.

3. List all the conditions that must be met to ensure that three events A, B, C are independent.

4. Suppose we toss two fair 6-sided dice. Let E_1 be the event that the first roll comes up 4. Let E_2 be the event that the sum of the two rolls is 4. Let E_3 be the event that the sum of the two dice is 7.

   (a) Are events E_1 and E_2 independent?

(b) Are events E_1 and E_3 independent?

(c) Are the three events independent?

Random variables

1. Draw a card from a standard deck of 52 cards and let the random variable X denote the value of the cards (where $X = 11$ for Jacks, 12 for Queens, 13 for Kings, and 14 for Aces).

   (a) What is the probability mass function f?

   (b) What is the the probability that a card is royalty (Jack, Queen, King or Ace)? Express this quantity in terms of the probability mass function and then compute it explicitly.

   (c) What is the probability of the event $3 \leq X \leq 7$?

2. Spinner #1 gives the values $1, 2, 3$ with equal probability and Spinner #2 gives the values $1, 2, 3, 4$ with equal probability. These outcomes are independent. We are interested in the random variable $X =$ (outcome of Spinner #1)$\times$(outcome of Spinner #2). What is the sample space? Find the range of X and determine the probability mass function f.

3. Flip a fair coin twice. What is the sample space? Let X be the number of heads. What is the probability mass function f?

4. Flip a fair coin n times, and let X denote the number of heads.

   (a) Find the probability mass function for $n = 3$.

   (b) Find the probability mass function for $n = 4$.

   (c) Find the probability mass function for $n = 5$.

5. A 6-sided die is rolled twice. What are the ranges of the following random variables?

   (a) $X =$ the minimum of the two rolls.

   (b) $Y =$ the maximum of the two rolls.

6. Assuming the die in the previous problem is fair, compute the probability mass function for the two random variables in parts (a)–(b).