UC Berkeley Math 10B, Spring 2016: Homework 3

Due: Wednesday, February 10

1. In how many ways can we distribute seven apples and six oranges among four children so that each child receives at least one apple?

2. In how many ways can one distribute 10 (identical) marbles among 6 distinct containers?

3. How many different terms are there in the expansion of $(x_1 + \cdots + x_7)^{25}$ after all terms with identical sets of exponents are combined?

4. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 = 21$$

   where the x_i are non-negative integers such that:

   - there are no restrictions?
   - $x_i \geq 1$ for $i = 1, \ldots, 5$?
   - $x_i \geq 2$ for $i = 1, \ldots, 5$?
   - $0 \leq x_1 \leq 10$, but no restrictions on x_2, x_3, x_4, x_5?

5. Construct a table of Stirling numbers to compute

   (a) $S(4, 2)$
   (b) $S(5, 4)$
   (c) $S(6, 3)$
   (d) $S(6, 4)$
   (e) $S(6, 5)$

   The idea is to use the recursive formula for Stirling numbers that's in the "textbook." Before using that formula, make sure that you understand how it's derived.

6. In how many ways can 5 different employees be assigned to 3 identical offices, if:

   (a) there are no restrictions?
   (b) each office must have at least one person?

7. How many ways are there to pack six identical DVDs into three indistinguishable boxes so that each box contains at least one DVD?

8. How many ways are there to distribute five balls into seven boxes, where each box must have at most one ball in it, if

   (a) both the balls and boxes are labeled?
   (b) the balls are labeled, but the boxes are unlabeled?
   (c) the balls are unlabeled, but the boxes are labeled?
   (d) both the balls and boxes are unlabeled?

9. How many ways are there to distribute five balls into three boxes, where each box must have at least one ball in it, if

- both the balls and boxes are labeled?
- the balls are labeled, but the boxes are unlabeled?
- the balls are unlabeled, but the boxes are labeled?
- both the balls and boxes are unlabeled?

10. Suppose you flip a fair coin three times.
    (a) Describe the relevant probability space Ω and determine $|\Omega|$.
    (b) Suppose you only report X = the number of heads. What is the probability of each possible outcome of X?

11. Suppose you roll a fair die 3 times.
    (a) Describe the sample space Ω and determine $|\Omega|$.
    (b) What is the probability that the sum of the three rolls is less than 5?
    (c) What is the probability that you roll at least one 1?
    (d) What is the probability that at least one of the two events described happens; that is, the probability that the sum is less than 5 or you roll a 1 (or both)?

12. What is the probability that when you roll a fair die 6 times, you never get an odd number?

13. Which is more likely: rolling a total of 9 when two dice are rolled or rolling a total of 9 when three dice are rolled?

14. What is the probability that a five-card poker hand contains no aces? (A five-card poker hand consists of 5 unordered cards from a 52 card standard deck.)

15. What is the probability that a five-card poker hand is a flush (all cards are the same suit)?

16. In the game of bridge, all 52 cards are dealt evenly to 4 players.
    (a) What is the probability that you are dealt all of the hearts?
    (b) What is the probability that you are dealt no hearts?
    (c) What is the probability that you are dealt a hand with no cards higher than a 9? (Such a hand is referred to as a *Yarborough*).

17. Suppose that a weather forecaster reports the following: P(rain today) = 0.30, P(rain tomorrow) = 0.40, P(rain both today and tomorrow) = 0.20, P(rain today or tomorrow) = 0.60. Define an appropriate sample space, write these expressions in terms of unions and intersections of subsets of the sample space, and assess if the statement is reasonable.

18. You flip a fair coin twice. What is the conditional probability that both flips are tails, given
    (a) the first flip is tails?
    (b) at least one of the two flips is tails?

19. What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1?

20. Consider the Monty Hall Problem but now with four doors. (For the original problem, see `https://www.youtube.com/watch?v=4Lb-6rxZxx0`.) There is still only one prize and the host still opens only one other door. What is the probability of winning if you
    (a) stay with your original choice?
    (b) switch doors, and choose randomly between the remaining two doors?