Math 10B, Spring 2016
UC Berkeley, Homework 2

Due: Wednesday, February 3

Once again, I am simply passing on problems that were assigned in a previous version of Math 10B. These problems seem completely reasonable to me, and I feel that everyone should learn how to solve problems of each type that's represented. Some of the problems are a bit long (multiple parts!)—sorry!

1. Let $S = \{1, 2, 3, 4\}$.

   (a) List all the 3-combinations of S.

   (b) List all the 3-permutations of S.

2. Compute

   (a) $P(6, 3)$ and $C(6, 3)$;

   (b) $P(6, 5)$ and $C(6, 5)$;

   (c) $P(8, 1)$ and $C(8, 1)$;

   (d) $P(10, 9)$ and $C(10, 9)$;

   (e) $P(8, 5)$ and $C(8, 5)$.

3. How many bit strings of length 12 contain:

   (a) exactly three 1s?

   (b) at least three 1s?

   (c) an equal number of 0s and 1s?

4. A regular six-sided die is rolled four times. How many possible outcomes

   (a) are there in total?

   (b) contain exactly two sixes?

   (c) have outcomes occurring in strictly increasing order?

5. A box contains 14 different colored balls.

   (a) We draw a ball at random, write down its color, and then replace it in the box. We do this 7 times in all. How many possible lists of colors are there?

   (b) Same question, except we do not replace the balls we have drawn.

6. The Applied Magic Department at Miskatonic University has 10 men and 15 women faculty members. How many ways are there to form a committee with six members, if it must have more women than men?

7. How many ways are there to deal hands of five cards to each of six players from a deck containing 52 different cards?

8. In how many ways can 12 different books be distributed among four children so that

   (a) each child gets 3 books?

(b) the two oldest children get 4 books each, and the two youngest get 2 books each?

9. If six people are seated about a round table, how many different circular arrangements are possible, if arrangements are considered the same when one can be obtained from the other by rotation?

10. How many ways are there to seat six boys and eight girls in a row of chairs so that no two boys are seated next to each other?

11. (a) Find the coefficient of $x^5 y^2$ in $(x + y)^7$.

    (b) Find the coefficient of $a^5 b^2$ in $(2a - 3b)^7$.

12. Show that for all $n \geq 2$,
$$\binom{n+1}{2} = \binom{n}{2} + n.$$

13. Show that
$$\binom{n}{k} = \frac{n}{k}\binom{n-1}{k-1}.$$

14. Prove that
$$\sum_{k=0}^{n} k(k-1)\binom{n}{k} = n(n-1)2^{n-2}.$$

15. Determine all real numbers r that satisfy the equation
$$\sum_{i=0}^{50} \binom{50}{i} 8^i = r^{100}.$$

16. In how many ways can four bones be distributed among three dogs? Here both the bones and the dogs may be distinguishable or not distinguishable. Discuss all four cases.