UC Berkeley Math 10B, Spring 2016: Homework 11

Due April 27

Gaussian elimination

1. Use row operations to convert the following augmented matrices $(A \mid b)$ into the form $(U \mid c)$, where U is upper triangular:

$$\text{(a)} \left(\begin{array}{ccc|c} 2 & 1 & 8 & 1 \\ -1 & 1 & -1 & 0 \\ -2 & 5 & 4 & 1 \end{array}\right) \quad \text{(b)} \left(\begin{array}{ccc|c} 4 & 5 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & 0 \end{array}\right) \quad \text{(c)} \left(\begin{array}{ccc|c} 3 & 2 & 1 & 1 \\ -1 & 4 & 1 & 4 \\ 2 & 1 & 3 & 1 \end{array}\right)$$

2. Find all solutions, if any, to the following systems of linear equations by putting them into augmented matrix form, using row operations to get the systems into upper triangular form, and then using back substitution:

$$\text{(a)} \begin{cases} 2x + 3y - z = 0 \\ x + 2y + z = 3 \\ x + 3y + 3z = 7 \end{cases} \quad \text{(b)} \begin{cases} x + y - 2w = 2 \\ x - y + 6z = 0 \\ 2y + 2z - 2w = 1 \\ 10x - 8y - 2z - 2w = 1 \end{cases} \quad \text{(c)} \begin{cases} x - 3y + z - w = 7 \\ 2x + y + w = 0 \\ 3y - z + w = -6 \end{cases}$$

3. Use row operations to find the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & -1 \\ -3 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}$$

Eigenvalues, eigenvectors, linear systems of differential equations

4. Find all the eigenvalues and eigenvectors for the matrices

$$\text{(a)} \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix} \quad \text{(b)} \begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix} \quad \text{(c)} \begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$$

5. Show that two solutions of the system of ODEs $y' = Ay$, where

$$A = \begin{pmatrix} 7 & -2 \\ 15 & -4 \end{pmatrix}$$

are given by

$$y^1 = (2e^{2t}, 5e^{2t})^T, \quad y^2 = (e^t, 3e^t)^T$$

Remark: The reason we write "y^1" and "y^2" instead of "y_1" and "y_2" is that if, for instance, $y = \left(\begin{smallmatrix} 5\sin t \\ 3\cos t \end{smallmatrix}\right)$, then "$y_1$" and "$y_2$" usually refer to "$5\sin t$" and "$3\cos t$".

6. For each of the following two systems of differential equations, find two real-valued functions y^1 and y^2 from $\mathbb{R}^2$ to $\mathbb{R}^2$ that are solutions and are not constant multiples of each other:

$$\text{(a)}\ y' = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} y \quad \text{(b)}\ y' = \begin{pmatrix} -1 & 3 \\ -3 & -1 \end{pmatrix} y$$

7. Solve the initial value problem

$$\begin{cases} y' = \begin{pmatrix} 1 & 2 \\ 6 & -3 \end{pmatrix} y \\ y(0) = (1, -1)^T \end{cases}$$

8. Conduct an internet search and find an interesting application of eigenvalues and eigenvectors in **biology**. Describe that application using your own words.