UC Berkeley Math 10B, Spring 2016: Homework 10

Due April 20

Matrix algebra

1. Let

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 3 & -1 \\ 0 & 2 \\ 1 & -1 \end{pmatrix}, \quad D = \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}.$$

Compute the following quantities or explain why they are undefined:

(a) $A - B$	(b) $2B^T$	(c) AB	(d) BA	(e) $A - C$
(f) A^TC	(g) CA	(h) $-D$	(i) DC	(j) CD

2. Consider the two matrices

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 0 & x \\ y & 1 \end{pmatrix}$$

Find all real values of x and y such that A and B commute, i.e.,such that $AB = BA$.

3. Consider the vectors $a = (1, -2, 2)$, $b = (2, 1, 0)$, and $c = (-1, -1, 1)$.

   (a) Find the norms $|a|$, $|b|$, and $|c|$.

   (b) Find the inner products $a \cdot b$, $a \cdot c$, and $b \cdot c$.

4. Which pairs of vectors $a = (a_1, a_2)$ and $b = (b_1, b_2)$ satisfy $|a \cdot b| = |a||b|$? In other words, when does equality hold in the Cauchy–Schwarz inequality for two vectors in $\mathbb{R}^2$?

Inverses and determinants

5. Consider the matrix $A = \begin{pmatrix} 3 & -5 \\ 1 & -2 \end{pmatrix}$.

   (a) Find the inverse A^{-1}.

   (b) Compute $A^{-1}A$ and AA^{-1} and confirm that $A^{-1}A = AA^{-1} = I$.

   (c) Use A^{-1} to solve the linear system $Ax = b$ where $b = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.

   (d) Compute Ax and confirm that $Ax = b$.

6. Compute the inverses of the following matrices, or explain why the inverse does not exist:

   (a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} -1 & 2 \\ 2 & -4 \end{pmatrix}$ (d) $\begin{pmatrix} 3 & 5 \\ 0 & 2 \end{pmatrix}$ (e) $\begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}$

7. Compute the determinants

   (a) $\begin{vmatrix} 1 & 0 & 2 \\ 0 & 3 & 1 \\ 4 & 1 & 1 \end{vmatrix}$ (b) $\begin{vmatrix} 4 & -3 & 2 \\ 1 & -3 & 1 \\ 9 & 1 & 0 \end{vmatrix}$ (c) $\begin{vmatrix} 6 & 7 & -2 \\ -2 & -3 & 1 \\ 7 & 7 & 1 \end{vmatrix}$

   (d) $|cI|$, where c is a scalar and I is the n-by-n identity matrix

8. (a) Show that the matrix $A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & -2 & -1 \\ 3 & 1 & 2 \end{pmatrix}$ is invertible.

   (b) Show that $A^{-1} = \begin{pmatrix} -3 & -2 & 2 \\ -5 & -4 & 3 \\ 7 & 5 & -4 \end{pmatrix}$.

   (c) Use A^{-1} to solve the linear system $Ax = b$, where $b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$

9. Show that $(A + B)^T = A^T + B^T$. More specifically, for any two matrices A and B, if one of $(A + B)^T$ and $A^T + B^T$ exists, then the other exists and the two are equal.