Math 10B, Spring 2016
UC Berkeley, Homework 1
Due: Wednesday, January 27

These problems are taken nearly verbatim from last year's class.

The Basics of Counting

1. Suppose that ten Republican and three Democratic candidates are candidates for President.
   (a) If the office holder is to be one of these candidates, how many possibilities are there for the eventual winner?
   (b) How many possibilities exist for a pair of candidates (one from each party) to oppose one another for the general election?

   In both cases, state which counting principles you use.

2. Suppose that 286 students are enrolled for Math 10B Lecture 1 and 287 students are enrolled for Math 10B Lecture 2.
   (a) In how many ways can we choose a pair of Math 10B students, one from each lecture?
   (b) In how many ways can we choose a pair of students from the first class?
   (c) In how many ways can we choose a pair of students from the second class?
   (d) In how many ways can we choose a pair of Math 10B students?

   Is the answer to (d) the sum of the answers to questions (a)–(c)?

3. A T-shirt store sells one particular style in 12 colors. This T-shirt has both male and female versions, and comes in three sizes for each sex. How many different types of this shirt are made, assuming all variations are sold?

4. How many DNA strings of length n, where n is a positive integer, end and start with an **A** or a **G**? (Each entry of a DNA string is either **A**, **C**, **G** or **T**.

5. How many license plates can be made using either four digits followed by three letters or four letters followed by three digits?

6. A *codon* is a sequence of length three over the *RNA alphabet*, which consists of the four letters $\mathbf{A}, \mathbf{C}, \mathbf{G}, \mathbf{U}$. How many codons are there? How many codons contain the letter **A** twice? How many codons consist of three distinct letters?

7. How many strings of four decimal digits
   (a) begin with an odd digit?
   (b) have exactly two digits that are 8s?

8. How many ways can we arrange the letters a, a, a, a, a, b, c, d, e so that no a is adjacent to another a?

9. (a) In how many ways can the letters in CELEBRATE be arranged?
   (b) How many of these arrangements have all three E's together?

10. Use a tree diagram to find the number of bit strings of length 4 without 2 consecutive 0s.

The Inclusion–Exclusion Principle

11. In a herd of 200 sheep, 34 are black and the others are white. There are 98 female sheep, and 15 of these are black. How many sheep are male and white?

12. Determine the number of integers between 1 and 1000 that are
    (a) not divisible by 3 or 5
    (b) not divisible by 3, 5, or 7

13. How many social security numbers (nine-digit sequences) have each of the digits 1, 3, and 7 appearing at least once?

The Pigeonhole Principle

14. Suppose that there are 2001 students enrolled in a huge online math class. Show that the class must have at least 1001 male students or at least 1001 female students.

15. Show that if there are over 100,000,000 wage earners in the United States who earned less than \$1,000,000 last year, then there are two who earned exactly the same amount of money, to the penny.

16. There are 51 houses on a street. Each house has a different address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers.

17. Let $ABCD$ be a square with $AB = 1$. Show that if we select five points in the interior of this square, there are at least two whose distance apart is less than $1/\sqrt{2}$.

18. Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers whose sum is 9.

19. Let S be a set of six positive integers whose maximum is at most 14. Show that the sums of the elements in all the nonempty subsets of S cannot all be distinct.