The remains of the course

Kenneth A. Ribet

Math 10A
November 30, 2017

This is the end

This is the last week of classes.

This is the last class. You may see Kate taking a few photos during this class. When we're through talking, we can line up on the steps of Dwinelle for an end-of-semester portrait.

RRR Week December 4–8:

This class will meet in this room on December 5, 7 for structured reviews by T. Zhu and crew.

I'll be out of town

During RRR week, I will be on the East Coast again. Accordingly, I won't be holding office hours next week.

Sorry!

I'll be back for exam week and will see you then.

I'll be out of town

During RRR week, I will be on the East Coast again. Accordingly, I won't be holding office hours next week.

Sorry!

I'll be back for exam week and will see you then.

I’ll be out of town

During RRR week, I will be on the East Coast again.
Accordingly, I won’t be holding office hours next week.

Sorry!

I’ll be back for exam week and will see you then.

Final exam
Thursday evening, December 14, 7–10PM.

If your GSI is Teddy Zhu or Freddie Huang, you are in 120 Latimer.

Otherwise you are in 1 Pimentel.

More on the exam

No devices or books are allowed. Each student can bring one two-sided standard-sized sheet of paper to the exam room, but otherwise the exam is closed-book.

The final exam is intended to be comprehensive, but it's easy to imagine that there will be more questions on the final third of the course (meaning the material discussed beginning October 24) more than on either of the first two thirds of the course.

Don't even think about submitting a solution with no words of explanation. Explanations pay a crucial role in possible partial credit for solutions that are correct in principle but flawed in some way. A "correct answer" with no supporting words and sentences is very unlikely to earn full credit.

To be informed of spring semester Faculty Club events, please send me email asking that your email address be added to my “spring dining” list.

Course evaluations

Please do them!

There’s a breakfast on Friday, but it’s fully subscribed.

Pop-in lunch on Friday at
High Noon (Faculty Club).

The rest of these slides contain problems for discussion. They will full into three categories:

1. Problems we have already discussed.
2. Problems we will discuss today for the first time.
3. Problems that we won't discuss but will be assigned for homework.

The next and last homework will be due on Tuesday, December 5. Your GSI will let you know how to get your homework paper to her or him.

A mileage test is conducted for a new car model, the "clunker." Thirty randomly selected clunkers are driven for a month, and the mileage for each car is measured. The mean mileage for the sample is 28.6 miles per gallon; the sample standard deviation is 2.2 miles per gallon. Estimate a 95% confidence interval for the mean miles per gallon in the entire population of clunkers.

This business of a "confidence interval" is new jargon.

To simplify, let's assume at first that the number 30 is large enough so that we can use the normal distribution instead of the Student distro with $\nu = 29$. We then have implicitly a random variable X with standard deviation $\sigma = 2.2$ and unknown mean μ. The sample mean is 28.6, so the number z is going to be

$$(28.6 - \mu)\frac{\sqrt{30}}{2.2}.$$

The 95% probability range for the normal variable coresonds roughly to $-2 \leq z \leq 2$. The probability that a standard normal variable lies between -2 and $+2$ is 0.9544, if you want more precision.

A calculation shows that $-2 \leq z \leq +2$ if and only if

$$27.8 \leq \mu \leq 29.4.$$

The "confidence interval" that's requested is then $[27.8, 29.4]$.

I put this problem on the last 10A homework assignment last year. The GSI who wrote up the solution used 1.96 in place of 2; note that I used the word “roughly” on the previous slide. The 2016 answer was then more precise, namely $[27.81, 29.39]$.

A more interesting question is getting the 95% probability range with the Student *t*-dsitribution and $\nu = 29$. Using the relevant calculator, I found that the analogue of 2 (or 1.96) is 2.045.

This makes sense because the *t*-distributions have fatter tails than the normal distribution.

Use the Central Limit Theorem to estimate the probability of getting heads 60 or more times when a fair coin is tossed 100 times.

Solution: we calculate $z = (.60 - .50)\dfrac{\sqrt{100}}{0.5} = 2$ and look up the probability in a table (or remember that it's about 2.5%). It's $0.5 - 0.4772 = 0.0228$ according to Table 7.3 on page 566.

The number 100 is far from infinity. The online Student t-test calculator that we used on Tuesday gives the probability as 0.241; remember that the tails are fatter than tails in the normal distribution when the sample size is small.

Use the Central Limit Theorem to decide which of the following two scenarios is the more likely one: getting 70 or more heads in 100 tosses of a fair coin or getting 600 or more heads in 1000 tosses of a fair coin.

This problem, or one close to it, was on the homework due two days ago. In each case, you calculate the z-score. The smaller of the two z-scores corresponds to the more likely event. Of course, we are comparing two different distributions by saying that they're each sufficiently like the normal distribution that they live on the same planet.

In other words, to do this problem, we are implicitly invoking the Central Limit Theorem.

Find all real numbers C such that $C\dfrac{e^x}{1+e^{2x}}$ is a PDF of a random variable. For each such C, find the corresponding CDF.

There's a unique C: it's the reciprocal of the number $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$. We can calculate the integral and we can find the CDF because we can find an antiderivative for $\dfrac{e^x}{1+e^{2x}}$, namely $\tan^{-1}(e^x)$. If I'm not mistaken, $C = \dfrac{2}{\pi}$.

HW#1

> *Suppose that X is a random variable whose PDF is $f(x)$ and that a is a real number. What is the PDF for the random variable $X - a$?*

I hope that everyone in the class can figure this out. Maybe we've already figured this out in class! I'll say no more today.

HW#2

Compute the expected value of a random variable whose PDF is 0 for $x < 1$ and $3x^{-4}$ for $x \geq 1$.

Same comment as before: no discussion right now. You should be able to check that this is a PDF, write down the corresponding CDF and do the indicated computation.

To determine the effectiveness of a certain diet in reducing the amount of cholesterol in the blood, 100 people are put on the diet. After they have been on the diet for some time, their cholesterol count will be taken. We will endorse the diet if at least 65 percent of the people have a lower cholesterol count after going on the diet. Assume that the diet has no effect on the cholesterol level. Use the Central Limit Theorem to estimate the probability that we will endorse the diet.

This is another CNT problem. We need to assume that a person's cholesterol count is a number computed to such high precision that a person's count can either go up or go down but can't stay the same. The "no effect" assumption means that a given count is equally likely to go up as to go down. In other words, you are essentially being asked to estimate the probability that 100 flips of a fair coin give you 65 or more heads. You can do that. **HW#3**

Find all values of a and b such that

$$p(t) = \frac{ae^{bt}}{1 + ae^{bt}}$$

is a solution to the logistic differential equation

$$p'(t) = p(1 - p).$$

This is a problem for homework. **HW#4**

HW#5

Let λ be a positive number. Suppose that the PDF of X is 0 for negative x and $\lambda e^{-\lambda x}$ for $x \geq 0$. Calculate the expected value and the variance of X.

This is a problem for homework.

HW#6

> *An inventor has developed a new, energy-efficient lawn mower engine. He claims that the engine will run continuously for 300 minutes on a gallon of gasoline. From his stock of 2000 engines, the inventor selects a simple random sample of 50 engines for testing. The engines run for an average of 295 minutes, with a sample standard deviation of 20 minutes. (The run times for the population of engines are normally distributed.) Test the null hypothesis that the mean run time is 300 minutes against the alternative hypothesis that the mean run time is not 300 minutes.*

This is a problem for homework. This is clearly intended to be a Student t-test problem. Use an online calculator with $\nu = 49$.