

Math 10A midterm exam, October 26, 2017

You put away all books, calculators, cell phones and other devices. You consulted a single two-sided sheet of notes. You wrote carefully and clearly, *USING WORDS* (not just symbols). The paper you handed in was your only representative when your work is graded.

Point counts:

Problem	1	2	3	4	5	6	7	Total
Points	5	5	6	6	6	6	6	40

**1.** The perimeter of a regular  $n$ -gon inscribed in the circle of radius 1 is  $2n \sin(\frac{\pi}{n})$ . Find the limit as  $n \rightarrow \infty$  of this expression. (Explain in words what you are doing—this requirement applies to each of the questions on this midterm.)

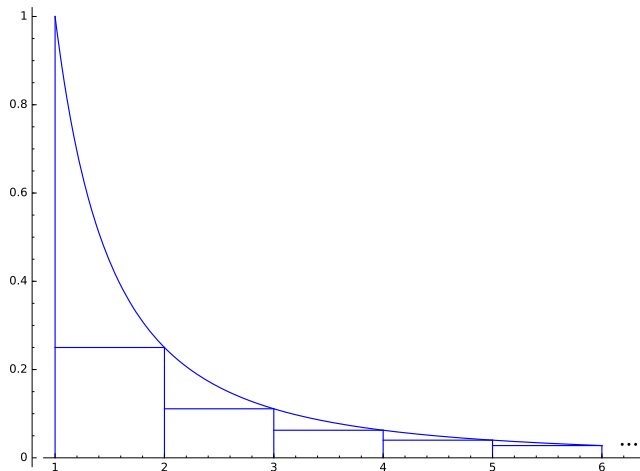
**2a.** Show that  $\frac{1}{m^2 + 3m + 2} = \frac{1}{m + 1} - \frac{1}{m + 2}$  for  $m \geq 0$ .

**b.** Find the sum of the infinite series  $\sum_{n=0}^{\infty} \frac{1}{n^2 + 3n + 2}$  by considering the partial sums of the series.

**3.** Referring to the diagram below, explain carefully why

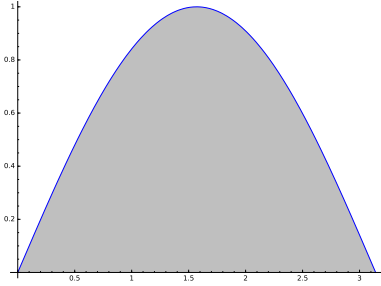
$$\frac{1}{2^2} + \frac{1}{3^2} + \cdots + \frac{1}{n^2} \leq 1 - \frac{1}{n}$$

for  $n \geq 2$ .



You acted with honesty, integrity, and respect for others.

4. Determine the volume of the solid obtained by revolving the area under  $y = \sin x$  from  $x = 0$  to  $x = \pi$  about the  $x$ -axis. [Hint: it may be helpful to know that  $\cos 2x = 1 - 2\sin^2 x$ .]



5. Find  $y$  as a function of  $x$ , given  $\frac{dy}{dx} = y(2x + 1)$  and  $y(0) = 2$ .
6. Use integration by parts, twice, to find an antiderivative of  $e^x \sin x$ .
7. Use the chain rule and the fundamental theorem of calculus to find

$$\frac{d}{dx} \left( \int_{x^2}^{x^3} \sin(t^2) dt \right).$$