

Professor Ken Ribet

Homework due Tuesday, November 28, 2017

From the book: review questions, pp. 588–589: 6, 9

1. Which is more likely: getting 60 or more heads in 100 tosses of a fair coin or getting 225 or more heads in 400 tosses of a fair coin?

2. If the continuous random variable X has PDF equal to $f(x)$, then we have

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x) dx$$

for all reasonable functions g . Use this information to calculate the expected value of $|X|$ when X is a standard normal variable (with mean 0 and standard deviation equal to 1).

3. Let b be a positive number and let $f(x) = \frac{b}{2}e^{-b|x|}$ for real numbers x .

a. Show that $f(x)$ is a PDF.

b. Find the mean of a random variable whose PDF is $f(x)$.

c. Express as an integral the standard deviation of a random variable whose PDF is $f(x)$. (Evaluate the integral if you can.)

d. Suppose that x_1, x_2, \dots, x_n form a random sample from a distribution whose PDF is $f(x)$. Find the maximum likelihood estimate of the parameter b .

4. Bob owns an aquarium with n fish (where n is unknown). One of the fish is red; the others are black. Bob behaves in the following bizarre way (remember: this is math homework). First he picks out a fish at random and observes that it is black. He returns this fish to the aquarium and adds eight more black fish (so now there are $n + 8$ fish). Finally, Bob picks out a fish at random and observes that it is red. Use the maximum likelihood method to obtain an estimate for n .