Math 10A Fall, 2017

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Homework due Thursday, November 16, 2017

Definitive version—unless there are errors that need to be corrected

Problems from the book:

- §7.1: 36
- §7.4: 37, 38
- Review questions, pp. 588–589: 3, 5, 10, 14, 18, 19

If the random variable X has $f(x)$ as its PDF, find the PDF of $37X$.

Let Ω be the set of the eight possible outcomes when we toss a coin three times:

$$\Omega = \{\, (\mathtt{000}), (\mathtt{001}), (\mathtt{010}), \ldots, (\mathtt{111}) \,\}.$$

If the coin comes up heads with probability p $(0 < p < 1)$ and tails with probabiity $q = 1 - p$, then the probability of $(\mathtt{000})$ is q^3, the probability of $(\mathtt{001})$ is $q^2 p$, and so on.

Let X_1, X_2 and X_3 be the functions that report the outcomes of the third respective tosses:

$$X_1(abc) = a, \quad X_2(abc) = b, \quad X_3(abc) = c.$$

Let $\overline{X} = \dfrac{X_1 + X_2 + X_3}{3}$ be the average of X_1, X_2 and X_3; let

$$Y = (X_1 - \overline{X})^2 + (X_2 - \overline{X})^2 + (X_3 - \overline{X})^2.$$

Find the expected value of Y as a function of p (and q).

Hint: Let's do the analogous computation there are two tosses instead of three. Then there are only X_1 and X_2; $\overline{X}$ is their average. The four outcomes `00`, `01`, `10` and `11` occur with respective probabilities q^2, pq, pq, p^2. (The sum of these probabilities, $q^2+pq+pq+p^2 = (p+q)^2$, is 1.) The values of $\overline{X}$ are respectively 0, $\frac{1}{2}$, $\frac{1}{2}$, 1. The values of $Y = (X_1 - \overline{X})^2 + (X_2 - \overline{X})^2$ are respectively 0, $\frac{1}{2}$, $\frac{1}{2}$, 0. The expected value of Y is $0 \cdot q^2 + \frac{1}{2} \cdot pq + \frac{1}{2} \cdot pq + 0 \cdot p^2 = pq$.

When there are three tosses, your answer should have been $2pq$. Can you guess what would happen if there were n tosses instead of 3?