

Professor Ken Ribet

Homework due Wednesday, December 7, 2016

This last homework assignment is due during RRR week. The official due date is December 7. Your GSI will let you know whether the homework is to be collected in an RRR discussion section on that day or will be collected otherwise. It can be handed in to your GSI on December 8 if that's your GSI's preference.

1. To determine the effectiveness of a certain diet in reducing the amount of cholesterol in the bloodstream, 100 people are put on the diet. After they have been on the diet for a sufficient length of time, their cholesterol count will be taken. The nutritionist running this experiment has decided to endorse the diet if at least 65 percent of the people have a lower cholesterol count after going on the diet. Assume that the diet has no effect on the cholesterol level. Use the central limit theorem to estimate the probability that the nutritionist will endorse the diet.

2. Find all values of a and b such that

$$p(t) = \frac{ae^{bt}}{1 + ae^{bt}}$$

is a solution to the logistic differential equation

$$p'(t) = p(1 - p).$$

3. If the continuous random variable X has PDF equal to $f(x)$, then we have

$$E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x) dx$$

for all reasonable functions g . Use this information to calculate the expected value of $|X|$ when X is a standard normal variable (with mean 0 and standard deviation equal to 1).

4. Suppose that the PDF of X is the function

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0, \\ 0 & \text{for } x < 0 \end{cases},$$

where λ is a positive constant. Calculate the expected value and the variance of X .

5. An inventor has developed a new, energy-efficient lawn mower engine. He claims that the engine will run continuously for five hours on a gallon of gasoline. From his stock of 2000 engines, the inventor selects a simple random sample of 50 engines for testing. The engines run for an average of 295 minutes, with a standard deviation of 20 minutes. (The run times for the population of engines are normally distributed.) Test the null hypothesis that the mean run time is 300 minutes against the alternative hypothesis that the mean run time is not 300 minutes. [Use the online calculator

<http://statistics.berkeley.edu/~stark/Java/Html/tHiLite.htm>
that is mentioned on page 154 of Prob-Stat.pdf.]

6. A random sample of 1562 undergraduates enrolled in marketing courses was asked to respond on a scale from 1 (strongly disagree) to 7 (strongly agree) to the proposition: “Advertising helps raise our standard of living.” The sample mean response was 4.27 and the sample standard deviation was 1.32. Decide whether or not to reject this (null) hypothesis: the mean μ for the full population of 1562 undergrads is 4.

7. A mileage test is conducted for a new car model, the “clunker.” Thirty randomly selected clunkers are driven for a month, and the mileage for each car is measured. The mean mileage for the sample is 28.6 miles per gallon; the sample standard deviation is 2.2 miles per gallon. Estimate a 95% confidence interval for the mean miles per gallon in the entire population of clunkers.