We begin a new course!

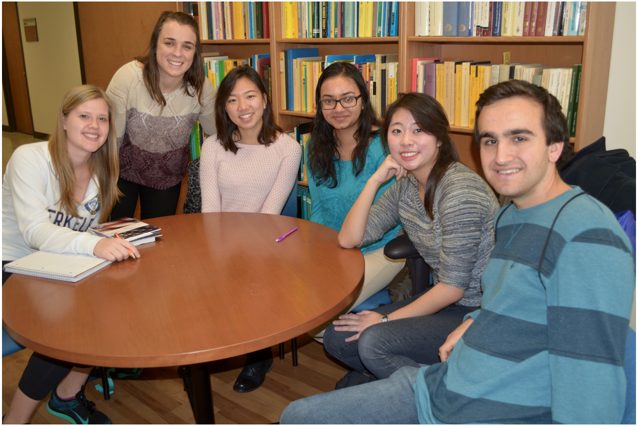
Kenneth A. Ribet

UC Berkeley

Math 10A
August 25, 2016

Welcome!

I hope that you are having a fine summer.



In Math 10A, the fun continues!!

Methods of Mathematics

This course discusses:

- Calculus (derivatives, intergrals,...)
- Differential equations
- Dynamics
- Probability

About me

I am

Ken Ribet (Professor Kenneth A. Ribet)

The best way to reach me is via email:

`ribet@berkeley.edu`

My office hours:

- Monday 2:10–3:10, 885 Evans
- Tuesday 10:30–noon, Student Learning Center
- Thursday 10:30–11:30, 885 Evans

I've been at UC Berkeley since forever (fall, 1978, actually). I am teaching this course because I was asked to teach Math 10B last semester and liked it so much that I insisted on teaching 10A this fall.

L&S1

In an effort to introduce myself further, I present these two links to you:

http://brcoe-review.s3.amazonaws.com/L%26S_W1_Guest_Lecture_Ken_Ribet_v03_segment1.mp4

http://brcoe-review.s3.amazonaws.com/L%26S_W1_Guest_Lecture_Ken_Ribet_v03_segment2.mp4

Together these constitute a video that I recorded in June, 2015 for students who were entering Berkeley in August, 2015. If I'm not mistaken, these videos were made available to your class (2020) this summer as well.

Numberphile

Numberphile is a YouTube channel that features math videos. It has over 10^6 subscribers. I have two videos on Numberphile:

https://www.youtube.com/watch?v=UTCScjoPymA
A "Stars and Bars" video about the *bagel problem* from Math 10B;

https://www.youtube.com/watch?v=nUN4NDVIfVI
An interview with me about Fermat's Last Theorem.

Discussion sections

Sections meet MWF for one hour:

Section	Time	Room	Instructor
101	8AM	205 Dwinelle	Vargas Pallete, Franco
102	8AM	110 Barker	Banks, Jess
103	9AM	87 Evans	Banks, Jess
104	5PM	250 Dwinelle	Lowengrub, Daniel
105	9AM	187 Dwinelle	Vargas Pallete, Franco
106	8AM	122 Barrows	Moini, Nima
107	4PM	118 Barrows	Robertson, James
108	Noon	81 Evans	Moini, Nima
109	1PM	3 Evans	Lowengrub, Daniel
111	2PM	81 Evans	McIvor, James
113	3PM	9 Evans	McIvor, James
114	3PM	B56 Hildebrand	Robertson, James
115	4PM	85 Evans	Zhu, Theodore
116	5PM	2 Evans	Zhu, Theodore

Our lectures

This room is equipped for “course capture,” meaning that we can record the lectures to a modest extent. See https://www.ets.berkeley.edu/services-facilities/course-capture for an explanation. This feature will provide you only with a faint shadow of what will go on in class. The reason is that I am likely to use the chalkboards as a major way to communicate information during the lectures. What I write on the chalkboard will *not* be recorded!

Don’t ditch class!

Ask questions during lectures. Lean in!

In fact, don’t be afraid to ask *dumb* questions. Other students have the same questions as you.

The Berkeley Registrar's office has a peculiar way of keeping time:

Recording Type: Computer Screen with Audio
Recording Time: Tuesday, Thursday 3:37pm-4:62pm
Publish Delay: As soon after capture as possible

The Registrar thinks that our class runs 3:30PM–4:59PM.

My view is that it runs 3:40PM–5:00PM. On most days, we'll take a three-to-five-minute break at some point during the lecture. If I get wrapped up in what I'm saying and you need a break, don't hesitate to interrupt me.

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Course web page

When I taught Math 10B last semester, we used the web site https://math.berkeley.edu/~ribet/10B/. Yesterday afternoon, I started building the web page https://math.berkeley.edu/~ribet/10A/. If you visit it, you'll see that most of it still pertains to Math 10B.

piazza

This class now has a `piazza` group, which you can undoubtedly locate through `https://piazza.com`. Most of you should have received an "invitation" to join the group yesterday afternoon.

The group facilitates comments, questions, online polls and many other things (which I have not used so far). Although you can post things anonymously, I've requested (in a `piazza` announcement) that you use your real names for routine posts (including "dumb" questions).

Mechanics of the class

Three exams:

- First midterm, September 22 (during class period)
- Last midterm, October 27 (during class period)
- Final exam 7–10PM, Friday night, December 16 (sorry!)

The two midterms are roughly at the 1/3 and 2/3 marks of the semester.

Important: the first MT comes right before the add/drop deadline and the second comes right before the deadline to change into or out of the P/NP grading option.

How you're graded

At the end of the semester, everyone gets a *composite grade* between 0 and 100:

- 20 points for the first MT;
- 20 points for the last MT;
- 35 points for the Friday night final;
- 10 points for homework;
- 15 points for quizzes.

The homework is intended to help you master the concepts and skills of this course. Your homework will be graded as follows: the solutions to two questions will be read carefully, and then there will be an overall check that you did most of the remaining problems.

Yes, you can work on homework with other students. It's in your interest to form study groups. Make friends with your classmates.

Right now!

Spend five minutes talking to the students on either side of you.

Exchange cell phone numbers.

Get to know your fellow students!

Final grades

I taught Math 10B last semester. Early on, I learned that Math 10B grades had been awarded as follows in previous versions of the course:

$$39\%\, A, \quad 37\%\, B, \quad 18\%\, C, \quad 6\%\, D/F.$$

I cloned this distribution last May. For our course, I will find out the historical Math 10A distribution and attempt to mimic it for you guys.

Important: it is hard to fail this course. The students with *D*s and *F*s are typically students who bailed on the course after the add/drop deadline and failed because they were enrolled formally but had dissociated themselves from the course.

About exams

In high school, you probably never saw an exam question that was not very, very similar to questions that you had seen in your textbooks, lectures and homework.

Now you're in the Big Leagues. The idea is that we want you to *think through* the concepts that we are studying and acquire the ability to own those concepts. Although exams are not the best places for creative thinking, we will ask you to do at least some thinking during our exams. (This is especially true for the final exam, which is longer than the MTs.)

Enrollment

If you are enrolled in this course, *Congratulations!*

If you are waitlisted in this course, you're in good shape.

If you want to enroll but can't even get on the waitlist, you're in less good shape.

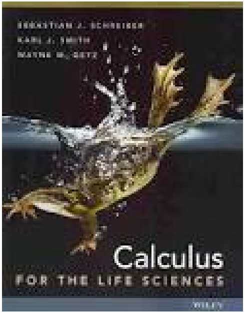
Personally, I have no power to enroll you. The Registrar has the ultimate power. The math department student services staff can sometimes help.

Experience suggests that everyone who wants to take this course will ultimately get in. *The math department does not turn away students.* At least that has been true up to now. Patience and courage are called for.

Textbooks

There are three .pdf files at your disposal (via bCourses): Differential_calculus, Integral_calculus, Prob-Stat. We will refer to those files from time to time.

More importantly, we are going to use the hardcopy textbook "Calculus for The Life Sciences" by Sebastian J. Schreiber, Wayne Getz, and William Smith.



This book costs over \$150 on Amazon. The "Berkeley custom edition" is available at the Cal Student Store for $\approx$ \$75. This is a good deal unless you can get the standard edition more cheaply.

What are your questions?

My teaching style?

I need your feedback to see how this class will work. When I write, there are three ways to go:

1. use the chalkboards;
2. lecture from prepared slides (a.k.a. Powerpoint, but mathematicians use LaTeX instead);
3. use the document camera.

The first method leaves no trace in the “capture” of the course but is the traditional way to communicate. Slides can be effective, but it’s hard to be flexible and to discuss things on the fly. The document camera is OK, but my handwriting is pretty bad when I write on a pad with a Sharpie.

Last semester there were no chalkboards in the room where I taught 10B. (There were a few whiteboards that students couldn’t really see.) We used methods #2 and #3 because #1 wasn’t available. I will try all possible methods and want to hear from you what you think is actually working.

The syllabus is ambitious and requires us to “cover” a great deal of material in each lecture. I will give the basic definitions and examples. In discussion sections, the GSIs will present further examples and challenge you with questions—typically intended for group work.

In the first two lectures, we will “cover” §§1.1–1.5 of the textbook. This material is intended as *review*.

First homework assignment

The first assignment will be due in discussion section on Wednesday, August 31. For this week, I will assign the exact same problems that my colleague Richard Bamler is assigning in the MWF 10A class:

- §1.1 7–9, 13, 18, 23, 27, 39
- §1.2 20, 22, 46
- §1.3 1, 25, 38
- §1.4 1–4, 35
- §1.5 1–10

There are a lot of problems, but this is *review*.

Functions

If I asked you to write down a function, you'd probably reply with a formula:

$$|x|, \quad \frac{1}{x^2-1}, \quad \frac{x+1}{(x+1)(x+3)}, \quad \sin x, \quad \lfloor x \rfloor, \quad \sqrt{x^2}, \cdots .$$

This is OK, but it's not perfect and probably isn't even great.

To define a function, you need to start by specifying its *domain* (the set where it's defined) and then say where the values are going. If the function is f and the domain is X, we write

$$f : X \to Y$$

if the values of f are intended to be in Y.

To define the function, you have to specify $f(x)$ for each $x \in X$. The specification can be done with a formula, a phrase, an algorithm or anything else that you can think of. What counts are the "outputs," the values $f(x)$ for x in the domain.

If two functions have the same domain and the same values, they are the *same function*. There are six functions on the previous slide; the first function and the sixth function are the *same*.

Note that the *floor function* $\lfloor x \rfloor$ is defined by a "rule": it's the integral part of a real number x. For example, $\lfloor -\pi \rfloor = -4$. See p. 124 of the Schreiber book for more info.

The *range* of f is the set of its values, i.e., the set of all $f(x)$ as x runs over X. In symbols this is

$$\text{range } f = \{ f(x) \mid x \in X \}.$$

If you just see a formula, you can guess at the intended domain. Your guess would be the largest set of numbers where the formula makes sense. For example the domain of

$$f(x) = \sqrt{x}$$

would "naturally" be the set of non-negative real numbers.

The domains of

$$\frac{x+1}{(x+1)(x+3)} \text{ and } \frac{1}{x^2-1}$$

would presumably be ____________ and ____________??
The bottom line, though, is that the domain has to be specified by the person who sets up the function.

Here's an example that looks forward to the definition of the derivative of a function. Let a be a real number (a "constant") and consider the quotient

$$\frac{(a+h)^2 - a^2}{h}$$

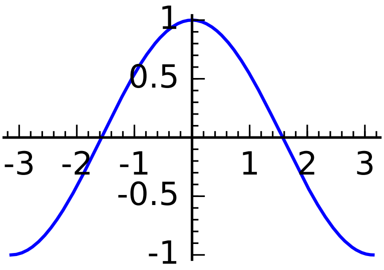
as a function of h. The fraction is

$$\frac{2ah + h^2}{h},$$

which we'd like to rewrite as $2a + h$. We can do that as long as we remember that the fraction is naturally defined for all real numbers $h \neq 0$, while the function of h given by the formula $2a + h$ is naturally defined for all real numbers h.

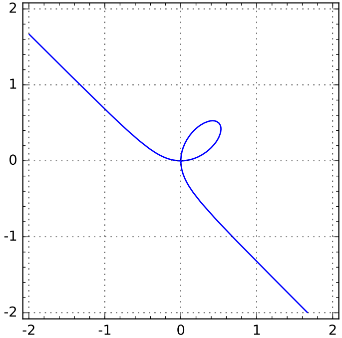
This is not a serious problem—we just have to pay attention. One function is defined for non-zero numbers h and the other for all numbers h. The two agree when they're both defined but they are *not the same function* because they have different domains of definition!

A function on a set of real numbers can be defined graphically: for each relevant x, you read off $f(x)$ from the graph.



Here, one might guess that the domain is the set of numbers from -3 to $+3$; actually we're looking at the graph of $\cos x$ from $-\pi$ to $+\pi$.

Not a function



Here we don't get a function: for certain x in the apparent domain there are two points on the graph with that value of x. When we're dealing with a function, the determination of $f(x)$ from a given value of x should not require flipping a coin. (You will have ample opportunities to flip coins in 10B.)

A function related to biology

Let X be the set of LifeFitness exercise machines in the RSF.

Let Y be the set of integers $0, 1, \ldots, 300$.

Let

$$f : X \to Y$$

be the function that assigns to $x \in X$ the maximum heart rate (measured in beats per minute) that x has recorded since it was installed in the RSF. (Most machines began recording Tuesday morning.) If x has not yet recorded anyone's heart rate, we set $f(x) = 0$.

This function is not defined by a "formula."

Power functions

The textbook singles out functions of the form

$$f(x) = ax^b$$

and calls them *power functions*. Fair enough!

Examples: $3x^2$, $5x^0 = 5$, $-17x^{-1} = \dfrac{-17}{x}$, $x^{1/6} = \sqrt[6]{x}$.

The first two functions are defined for all real numbers x, but x^{-1} is defined only for non-zero x and the sixth root of x can be defined only for non-negative x.

Exponential functions

Here we take a positive real number a and consider

$$f(x) = a^x, \quad x \in \mathbf{R}.$$

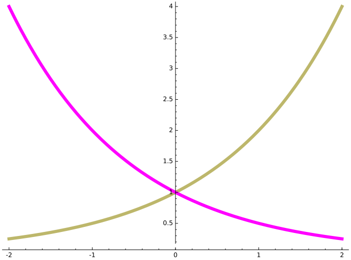
Famous values of a include $a = 10$, $a = e$ (whatever e is—it's about 2.71828) and $a = 2$. The symbol "**R**" stands for the set of all real numbers.

If $a = 1$, then $f(x)$ is the constant function 1, which really isn't an exponential function. The book rules out the choice $a = 1$, so let's do that as well.

Then we have the two cases

$$0 < a < 1, \qquad 1 < a.$$

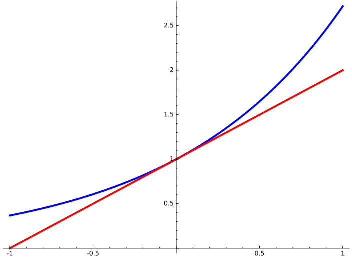
These are different in nature as we can see from the graphs of 2^x (darkkhaki) and $(1/2)^x$ (magenta):



Because $(1/2)^x = 2^{-x}$, the graphs are mirror images of each other. As x gets big and positive, the magenta curve approaches the x-axis. As x gets big and negative, the dark khaki curve approaches the x-axis.

As far as $e \approx 2.718281828459045$ goes, there are several possible (equivalent) definitions that one can use for this constant. The definition given on page 54 of our textbook is:

$$e = \lim_{n\to\infty} (1 + \frac{1}{n})^n.$$



An alternative characterization (as we'll see) is that the (blue) graph of $y = e^x$ kisses the (red) line $y = x + 1$ at the point $(0, 1)$ where it crosses the y-axis. The appeal of the line in question is that it has slope equal to 1.