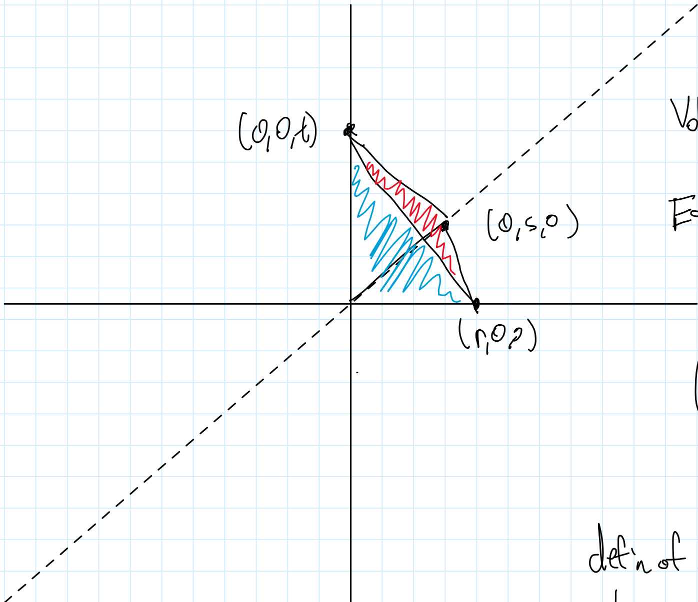


Volume $= \frac{1}{3} b\cdot h = \frac{1}{3}\cdot\left(\frac{r\cdot s}{2}\right)\cdot t = \frac{1}{6} r s t$

Equation of the plane: $\frac{x}{r}+\frac{y}{s}+\frac{z}{t}=1$

HM $\le$ GM $\qquad \frac{1}{i}=\frac{x}{r},\ \frac{1}{j}=\frac{y}{s},\ \frac{1}{k}=\frac{z}{t}$

$$\left(\frac{i^{-1}+j^{-1}+k^{-1}}{3}\right)^{-1} = \frac{3}{\frac{1}{i}+\frac{1}{j}+\frac{1}{k}} \le \sqrt[3]{ijk}$$

$$3 = \frac{3}{1 = \frac{x}{r}+\frac{y}{s}+\frac{z}{t}} \le \sqrt[3]{\frac{r}{x}\cdot\frac{s}{y}\cdot\frac{t}{z}}$$

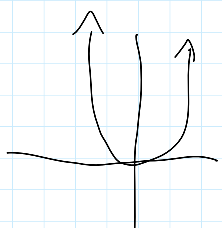
$$27 \le \frac{rst}{xyz}$$

def'n of convex

$$f\left(\frac{a+b}{2}\right) \le \frac{f(a)+f(b)}{2}$$

$f(x)=x^4$

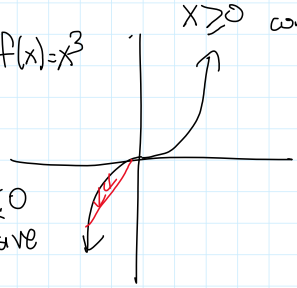
convex



$$\left(\frac{a+b}{2}\right)^4 \le \frac{a^4+b^4}{2}$$

$$P_1 = AM = \frac{a+b}{2} \le \sqrt[4]{\frac{a^4+b^4}{2}} = P_4$$

5b) Apply JI to $f(x)=x^3$



If $a, b < 0$

$$f\left(\frac{a+b}{2}\right) \ge \frac{f(a)+f(b)}{2}$$

$$\left(\frac{a+b}{2}\right)^3 \ge \frac{a^3+b^3}{2} \Rightarrow -\left(\frac{c+d}{2}\right)^3 \ge -\frac{c^3+d^3}{2}$$

$a=-c$
$b=-d$

$$\overline{abc} = 100\cdot a + 10\cdot b + c \equiv 1\cdot a + 1\cdot b + c \pmod 9$$

$$\overline{123} \equiv 1+2+3 = 6 \pmod 9$$

$$2^{29} = \overline{abcdefghi} \equiv a+b+c+d+\cancel{e+f}+g+h+i = S \pmod 9$$

$T - S =$ missing #

$$0+1+2+3+4+5+6+7+8+9 = T$$

$$45 - 2^{29} \equiv -2^{29} \pmod 9$$

$$\equiv -5 \equiv 4$$

Find last 2 digits of $123^{3^{80020}} \ne \cancel{(123^3)^{80020}}$

$$\phi(100) = \phi(2^2\cdot 5^2) = \#\{a \le 100 \text{ and } 2 \nmid a \text{ and } 5 \nmid a\}$$

Find last 2 digits of $123^{(3^{80020})} \neq \cancel{(123^3)^{80020}}$ $\equiv -5 \equiv 4$

$12345 = 123\cdot 100 + 45 \equiv 45 \pmod{100}$

$123^{(3^{80020})} \equiv \ ?? \pmod{100}$

ETT: $a^{\phi(d)} \equiv 1 \pmod{d}$ if $(a,d)=1$

Since $(123,100)=1$ and $\phi(100)=40$

$\boxed{123^{40} \equiv 1 \pmod{100}}$

So now want $3^{80020} \equiv ? \pmod{40}$

ETT: Since $(3,40)=1$ and $\phi(40)=16$

$3^{16} \equiv 1 \pmod{40}$ $80020 \equiv 80000+20 \equiv 0+4 = 4 \pmod{16}$

$\Rightarrow 3^{80020} \equiv 3^4 = 81 \equiv 1 \pmod{40}$

$123^{(3^{80020})} \equiv 123^1 = 123 \equiv \boxed{23} \pmod{100}$

$\phi(100) = \phi(2^2\cdot 5^2) = \#\{a \le 100 \text{ and } 2\nmid a \text{ and } 5 \nmid a\}$

$= 100 - (\#\{\text{div by } 2\} + \#\{\text{div by } 5\} - \#\{\text{div by 2 and } 5\})$

$= 100 - \left(\left\lfloor \frac{100}{2} \right\rfloor + \left\lfloor \frac{100}{5} \right\rfloor - \left\lfloor \frac{100}{10} \right\rfloor\right)$

$= 100 - (\underbrace{50+20-10}_{60}) = 40$

$\phi(40) = \phi(2^3\cdot 5) = 40 - \left(\left\lfloor \frac{40}{2} \right\rfloor + \left\lfloor \frac{40}{5} \right\rfloor - \left\lfloor \frac{40}{10} \right\rfloor\right)$

$= 40 - (20+8-4) = 16$

$123^{40} \equiv 1 \pmod{100}$

$3^{80020} = 40q + r$

$123^{(3^{80020})} = 123^{40q+r} = (123^{40})^q \cdot 123^r$

$\equiv 1^q \cdot 123^r = 123^{\boxed{r}} \pmod{100}$

$f(x) = 3x+1-\cos(x)$ has at most 1 real root.

Proof by contradiction.

Assume for contradiction we have 2 real roots $r<s$. $\Rightarrow f(r)=f(s)=0$.

Use RT on $[r,s]$

at least 1 real root

IVT $f(0)=0$ ✓

$f(-10)<0$

$f(10)>0$

$\binom{1}{1} + \binom{2}{1} + \binom{3}{1} = \binom{4}{2}$

Prove $\binom{k}{k} + \binom{k+1}{k} + \cdots + \binom{n}{k} = \binom{n+1}{k+1}$ $k=1$, $n=3$

RHS = # teams of k+1 people out of n+1 people

1, 2, 3, ... , n+1

Who is the first mouse chosen? →

Choose #1, need k more people out of n $\Rightarrow \binom{n}{k}$ +

Choose #2, need k more mice out of $n-1 \Rightarrow \binom{n-1}{k}$ +

Choose #3, need k more / $n-2 \Rightarrow \binom{n-2}{k}$

+ ⋮ +

$\binom{k}{k}$

} LHS

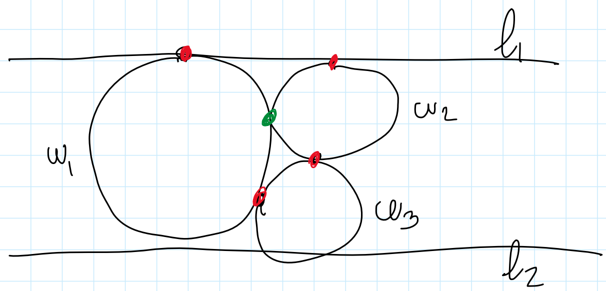
$\binom{0}{0}$

$\binom{1}{0}\ \binom{1}{1}$

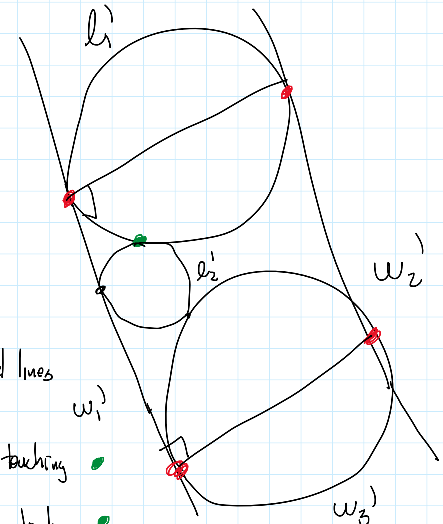
$\binom{2}{0}\ \binom{2}{1}\ \binom{2}{2}$

$\binom{3}{0}\ \binom{3}{1}\ \binom{3}{2}\ \binom{3}{3}$

$\binom{4}{0}\ \binom{4}{1}\ \binom{4}{2}\ \binom{4}{3}\ \binom{4}{4}$



$\Rightarrow$

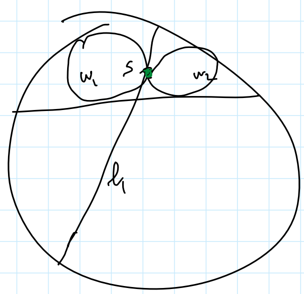


$\omega_1, \omega_2 \Rightarrow$ parallel lines

$l_1 \Rightarrow$ circle touching •

$\omega_3 \Rightarrow$ circle

$l_2 \Rightarrow$ circle touching •



$\Rightarrow$ w_1', w_2', l_1' parallel lines