Math 10A
Homework #6; Due Friday, 7/13/2018
Instructor: Roy Zhao

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1. Check that $F(x) = \sin(x) - x\cos(x)$ is an antiderivative of $f(x) = x\sin(x)$. Find all of the antiderivatives of f.

2. There is only one antiderivative of x^2 on $(-\infty, \infty)$ that is 3 when $x = 0$. Find it.

3. Suppose that f is a function for which $f'(x) = 2x + 1$ and $f(0) = 3$. What is $f(1)$?

4. You're filling a bucket of water. The rate at which you pour water into the bucket t seconds after you start is $r(t) = 1 + t$ liters/second for $0 \le t \le 5$. If the bucket is completely full after 5 seconds and started out empty, what is the capacity of the bucket?

5. True False If F is an antiderivative of f on (a, b), then $F'(x) = f(x)$ for all $x \in (a, b)$.

6. True False If F is an antiderivative of f on (a, b), then $F(x) + 3$ is also an antiderivative of f on (a, b).

7. True False Every function is its own antiderivative.

8. True False To calculate the definite integral $\int_{-5}^{5} \sqrt{25 - x^2}\, dx$, we must find an antiderivative of $\sqrt{25 - x^2}$ and use the FTC to evaluate it at the ends of the interval $[-5, 5]$.

9. True False We can only split an integral along its interval as in $\int_a^b f(x)\, dx = \int_a^c f(x)\, dx + \int_c^b f(x)\, dx$ only when c is between a and b.

10. Evaluate each of the following integrals using the fundamental theorem of calculus.

    (a) $\displaystyle\int_{-1}^{1} x^2 - x\, dx$

    (b) $\displaystyle\int_{0}^{2} (\sqrt{x} - 1)\, dx$

    (c) $\displaystyle\int_{1}^{2} (\frac{1}{x^2} + e^x)\, dx$

    (d) $\displaystyle\int_{0}^{1} \frac{dx}{1 + x^2}$

    (e) $\displaystyle\int_{0}^{\pi/2} \sin(x)\, dx$

11. Compute $\displaystyle\frac{d}{dt}\int_{1/t}^{t^3} e^{-x^2}\, dx$.

12. What is $\dfrac{d}{dx}\displaystyle\int_0^{f(x)} f(t)\,dt$?

13. True False $\dfrac{d}{dx}\displaystyle\int_0^1 f(t)\,dt = f(x)$.

14. True False $\displaystyle\int_0^1 f'(x)\,dx = f(1) - f(0)$.

15. True False If f and g are two functions, then

$$\int_a^b \frac{f(x)}{g(x)}\,dx = \frac{\displaystyle\int_a^b f(x)\,dx}{\displaystyle\int_a^b g(x)\,dx}.$$

16. Let $f : [0,3] \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} x & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } 1 \leq x \leq 2 \\ 3 - x & \text{if } 2 \leq x \leq 3 \end{cases}$$

Compute $\displaystyle\int_0^3 f(x)dx$ by drawing the graph and using formulas from geometry.

17. Given that

$$\int_0^2 f(x)\,dx = 3, \qquad \int_0^5 f(x)\,dx = 9, \qquad \int_1^5 f(x)\,dx = 7$$

find

(a) $\displaystyle\int_0^1 f(x)\,dx$

(b) $\displaystyle\int_0^2 3f(x)\,dx$

(c) $\displaystyle\int_1^2 (2f(x)+1)\,dx$

18. True False If $0 \leq f(x) \leq 10$ for all $x \in [0,1]$, then $0 \leq \displaystyle\int_0^1 f(x)\,dx \leq 10$.

19. Compute

(a) $\displaystyle\int 2dx$

(b) $\displaystyle\int (5x+3)^6\,dx$

(c) $\int 2t^2(t^3+1)^3\,dt$

(d) $\int \sqrt{3x+2}\,dx$

(e) $\int t^2(t^3+7)^{-1/2}\,dt$

(f) $\int \sin(2x-1)\,dx$

(g) $\int \sin^5(x)\cos(x)\,dx$

(h) $\int \frac{\sin(x)}{\cos^4(x)}\,dx$

(i) $\int_1^9 \frac{(\sqrt{x}+1)^3}{\sqrt{x}}\,dx$

(j) $\int_0^1 (x+1)\sin(x^2+2x+1)\,dx$

(k) $\int_1^9 (1+\frac{1}{t})^4\frac{1}{t^2}\,dt$

20. Label each of the following as even, odd, both or neither.

(a) $f(x)=|x|$

(b) $|\cos(x)|$

(c) $f(x)=\sin(x^3)$.

(d) $f(x)=x^2\sin(x)$

(e) $f(x)=x^2+1$

(f) x^2+x

(g) $f(x)=\arctan(x)$

21. Compute $\int_{-4}^4 x^3\cos(x)\,dx$.

22. Compute

(a) $\int xe^x\,dx$

(b) $\int x\cos x\,dx$

(c) $\int xe^{-4x}\,dx$

(d) $\int e^x\sin x\,dx$

(e) $\displaystyle\int_1^e \ln x \, dx$

(f) $\displaystyle\int_1^e \frac{\ln(2x)}{x^2} \, dx$