Math 10A
Homework #4; Due Tuesday, 7/3/2018
Instructor: Roy Zhao

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1. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

2. At which points on the curve $y = 1 - 40x^3 - 3x^5$ does the tangent line have the largest slope?

3. You have a cylinder whose height is equal to its radius that contains some volume of water. The rate at which the water evaporates from the cone is proportional to the exposed surface area of the water. Write down a formula for $\frac{dV}{dt}$ in terms of V and constants.

4. The radius of a spherical ball is increasing at a rate of $2cm/min$. At what rate is the surface area of the ball increasing with the radius is $8cm$?

5. A plane flying horizontally at an altitude of 1 mile and a speed of 500 mph passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 miles away from the station?

6. Two resistors are connected in parallel and their total resistance R can be given in terms of their individual resistance R_1, R_2 by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}.$$

   R_1 is increasing at a rate of $0.3\Omega/s$ and R_2 is decreasing at a rate of $0.2\Omega/s$. How fast is R changing with $R_1 = 80\Omega$ and $R_2 = 100\Omega$? Be sure to say whether R is increasing or decreasing.

7. A kite is flying at a current altitude of $100m$. The kite slowly flies further and further away as the string length increases at a rate of $3cm/s$. Assuming the altitude does not change, how fast horizontally is the kite moving when the angle the string forms with the ground is $\pi/6$?

8. A snowball melts so that its surface area decreases at a rate of $1cm^2/min$. Find the rate at which the diameter decreases when the diameter is 10 cm.

9. Find the following limits:

   (a) $\lim_{x \to 3} \dfrac{x - 3}{x^2 - 9}$

   (b) $\lim_{x \to 0} \dfrac{\tan(3x)}{\sin(2x)}$

   (c) $\lim_{\theta \to \pi} \dfrac{1 + \cos(\theta)}{1 - \cos(\theta)}$

(d) $\lim_{x\to\infty} \dfrac{x+x^2}{1-2x^2}$

(e) $\lim_{x\to 1} \dfrac{\ln x}{x}$

(f) $\lim_{x\to\infty} \dfrac{\ln\sqrt{x}}{x^2}$

(g) $\lim_{t\to 1} \dfrac{t^8-1}{t^5-1}$

(h) $\lim_{x\to 0} \dfrac{e^x-1-x}{x^2}$

(i) $\lim_{x\to 0} \dfrac{\sin(x)-x}{x^3}$

(j) $\lim_{x\to 0} \dfrac{x-\sin(x)}{x-\tan(x)}$

(k) $\lim_{x\to\infty} x\sin(\pi/x)$

(l) $\lim_{x\to 0} \csc(x)-\cot(x)$

(m) $\lim_{x\to\infty} (x-\ln x)$

(n) $\lim_{x\to-\infty} x\ln(1-1/x)$

(o) $\lim_{x\to\infty} x^{e^{-x}}$

10. True False If $\lim_{x\to c} \dfrac{f'(x)}{g'(x)} = L$, then $\lim_{x\to c} \dfrac{f(x)}{g(x)} = L$.