Math 10A
Homework #3; Due Friday, 6/29/2018
Instructor: Roy Zhao

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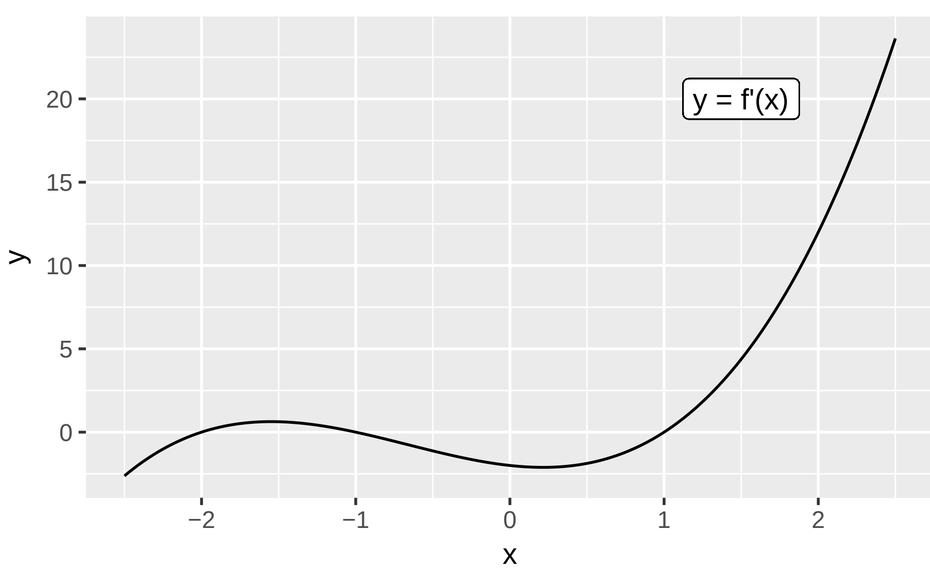
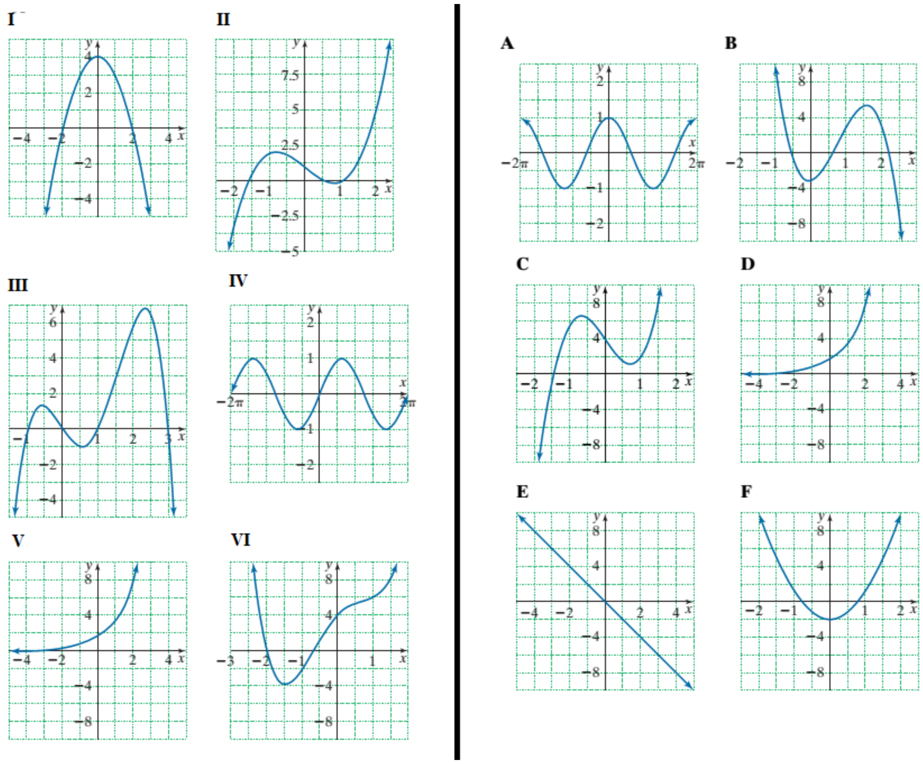
1. Give a formula for $\frac{dy}{dx}$ in terms of x and y using implicit differentiation.
   (a) $3x^2 - xy^2 + 2y^3 = 3$
   (b) $e^y \sin(x) = x^3 - y$
   (c) $\cos(x) = y^2 + x^3$
2. Find the equation of the tangent line to these functions at the given point.
   (a) $xy = 25$; $(5, 5)$
   (b) $\dfrac{1}{x} = \dfrac{1}{y} = \frac{1}{2}$; $(6, 3)$
   (c) $y^2(6 - x) = x^3$; $(3, 3)$
3. For each function below, compute $f''(x)$.
   (a) $f(x) = e^{x+2}$
   (b) $f(x) = \cos(3x - 1)$
   (c) $f(x) = \sin(x) - \ln(x^2)$
4. The position of a particle at time t is given by the function $f(t) = \sin(\pi t)$. Briefly describe the motion of the particle. What is the particle's acceleration at time $t = 3$?
5. Using the graph of $f'(x)$ given, find the intervals on which f is increasing and the intervals on which f is decreasing. What are the critical points of f? Which, if any, are local extrema? Label each local extreme point as a local maximum or a local minimum.



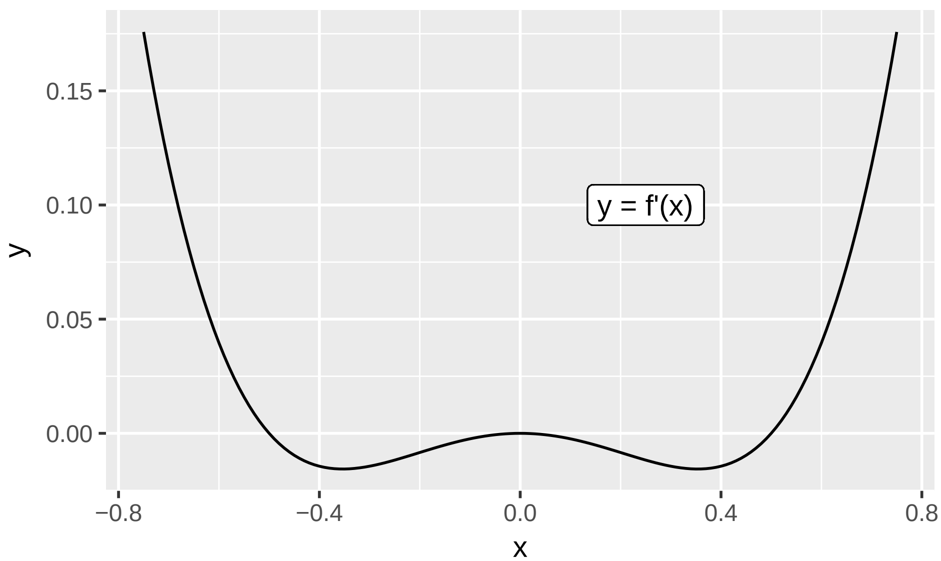
6. Match each graph on the left side to its derivative on the right.



7. Find the critical points, local maxima, and local minima of the given function on the given interval.

   (a) $\sin(x)$; $(-\pi, \pi)$
   (b) $3x^3 - 2x^2$; $(-2, 2)$
   (c) e^x; $(0, \infty)$
   (d) $10 + 6x - x^2$; $(-\infty, \infty)$
   (e) e^{t^2-2t+1}; $(-\infty, \infty)$
   (f) $\frac{x}{1+x}$; $(-\infty, \infty)$
   (g) $1 - e^{-x^2}$; $(-\infty, \infty)$

8. The graph below shows that $f'(0) = 0$, so $x = 0$ is a critical point of f. What is the second derivative of f at $x = 0$? What can you conclude from the second derivative test? Is $x = 0$ a local minimum, a local maximum, both, or neither?



9. For each of the following, sketch the graph of a function f satisfying the given requirements.
   (a) f is increasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$
   (b) f is concave down on $(-\infty, 0)$ and concave up on $(0, \infty)$.
   (c) f is increasing, concave up, and has domain $(0, \infty)$.
   (d) f is concave down on $(-\infty, \infty)$ and has a local maximum at $x = 2$.
   (e) f' is strictly increasing on $(-\infty, \infty)$.

10. True False If the derivative of f is increasing on the interval (a, b), then f is increasing on the interval (a, b).

11. True False If f is twice differentiable and concave down on the interval (a, b), then every critical point of f on the interval (a, b) is a local maximum.

12. For each of the following functions, sketch the graph of the function by finding all the relevant information (intercepts, asymptotes, intervals of increase/decrease, concavity, etc.)
   (a) $f(x) = \dfrac{x}{1 + x^2}$
   (b) $f(x) = \dfrac{1}{x - 1} + x$
   (c) $f(x) = (x^2 - 1)^3$
   (d) $f(x) = x^{1/5}(4 - x)$
   (e) $f(x) = \dfrac{2 + x}{1 + x}$

13. Sketch the graph of a function f satisfying all of the following conditions (there are many possible answers):

(i) f has a vertical asymptote at $x = 2$;

(ii) f is increasing on $(0, 2)$

(iii) f is concave up on $(2, \infty)$

(iv) f is not differentiable at $x = -1$

(v) f has a horizontal asymptote $y = 3$

14. True False If $x = 0$ is the location of a local maximum of f, then $f(0)$ is the global maximum of f on $(-1, 1)$.

15. True False If x is the location of a global minimum of a differentiable function f on an interval $[a, b]$, then $f'(x) = 0$.

16. True False If $f''(x) < 0$ on the interval (a, b), then the global minimum of f on $[a, b]$ occurs at either $x = a$ or $x = b$.

17. True False When a differentiable function on a closed interval has only one critical point, then the function is bound to have a global maximum or minimum there.

18. For each of the following functions, find the global extrema of that function on the given interval.

    (a) $3x - 2$; $[1, 4]$

    (b) $(x^3 + 1)^2$; $[-1, 1]$

    (c) $\cos(\pi x)$; $[-0.5, 0.5]$

    (d) $|x|$; $[-5, 5]$

    (e) $x + \frac{1}{x}$; $(0, \infty)$

    (f) xe^{-x}; $(-\infty, \infty)$

19. Find two nonnegative numbers x, y whose sum is 75 and such that xy^2 is as large as possible.

20. You want to fence off a rectangular region and you have 100m of fencing. What should the dimensions (the width and the length) of your region be so that the region has the largest possible area and uses all 100m of fencing?

21. A bacteria population is given by the formula $P(t) = \dfrac{6000t}{60 + t^2}$ where t is measured in days. When will the bacteria population reach its maximum size?

22. A wire that is $100cm$ long is cut into two pieces. One piece will be used to create a square and the other piece will be used to create a circle. How long should these pieces be so that the sum of the area of the square and circle is largest?

23. Find the point on the graph $y = 1 - x^2$ that is closest to the origin.