Math 10A
Homework #13; Due Tuesday, 8/7/2018
Instructor: Roy Zhao

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1. True False An eigenvector can be the zero vector.
2. True False To find the eigenvectors of a matrix, we need to find the eigenvalues first.
3. True False If λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2.
4. True False To find an eigenvalue, we set $\det(A-\lambda I) = 0$ because we want a nonzero solution to $(A - \lambda I)\vec{v} = \vec{0}$.
5. Find the eigenvalues and eigenvectors of the following matrices:
   (a) $\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$
   (b) $\begin{pmatrix} 2 & 3 \\ 0 & -1 \end{pmatrix}$
6. Find the eigenvalues of $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{pmatrix}$.
7. Solve the equation $\vec{y}' = A\vec{y}$ with $A = \begin{pmatrix} 1 & 2 \\ 6 & -3 \end{pmatrix}$ and $\vec{y}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.
8. Find the general solution to $\vec{y}' = A\vec{y}$ with $A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$.
9. (a) Verify that $\vec{y}_1(t) = \begin{pmatrix} 2e^{2t} \\ 5e^{2t} \end{pmatrix}$ and $\vec{y}_2(t) = \begin{pmatrix} e^t \\ 3e^t \end{pmatrix}$ are solutions to $\vec{y}' = A\vec{y}$ where $A = \begin{pmatrix} 7 & -2 \\ 15 & -4 \end{pmatrix}$.
   (b) What are the eigenvalues and eigenvectors of A?
10. Suppose that $\vec{y}_1(t) = e^{-3t}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\vec{y}_2(t) = e^t\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ are solutions to $\vec{y}' = A\vec{y}$. What are the eigenvalues and eigenvectors of A?
11. The characteristic polynomial of recurrence relations are related to the characteristic polynomial of a matrix. We will show this with this problem. Consider the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$.
    (a) What is the characteristic polynomial of the recurrence relation?

(b) Let $\vec{v}_n = \begin{pmatrix} a_{n-1} \\ a_n \end{pmatrix}$. Find a matrix A such that $A\vec{v}_{n-1} = \vec{v}_n$.

(c) What is the characteristic polynomial of the matrix A found above? How does your answer compare to your answer from (a)?

12. The characteristic polynomial of second order DEs are related to the characteristic polynomial of a matrix. We will show this with this problem. Consider the DE $y'' - y' - 2y = 0$.

    (a) What is the characteristic polynomial of the DE? What is the general solution to the DE?

    (b) Let $\vec{y}(t) = \begin{pmatrix} y(t) \\ y'(t) \end{pmatrix}$. Find a matrix A such that $\vec{y}' = A\vec{y}$.

    (c) What is the characteristic polynomial of the matrix A found above? How does this compare to your answer from (a)?

    (d) Find the general solution to $\vec{y}' = A\vec{y}$. How does this general solution compare to your answer from (a)?

13. True False The matrix $A^T A$ is always a square matrix.

14. True False If we use more data points, the overall error may increase yet the line may be a better predictor.

15. The following data is the HIV concentration in the blood of 5 individuals before and 6 months after a specific treatment for HIV infection.

Before	7.4	5.1	6.9	7.2	1.4
After	3.7	2.6	3.4	3.6	0.7

    1. Find the line of best fit in terms of $y = ax + b$.
    2. Given your value of a, is this treatment successful?
    3. Use your line of best fit to make a prediction for the blood HIV levels of a patient 6 months later if he starts at a concentration of 6.0.