Math 10A
Homework #12; **Due Friday, 8/3/2018**
Instructor: Roy Zhao

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1. True False Two vectors are perpendicular if and only if their dot product is 0.

2. True False We can add and subtract matrices only if they have the same size.

3. Let

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix}, B = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & 1 \end{pmatrix}, C = \begin{pmatrix} 3 & -1 \\ 0 & 2 \\ 1 & -1 \end{pmatrix}, D = \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix},$$

$$\vec{v} = \begin{pmatrix} 3 & -1 \end{pmatrix}, \vec{w} = \begin{pmatrix} 4 & 2 \end{pmatrix}, \vec{u} = \begin{pmatrix} 3 & -1 & 5 \end{pmatrix}, \vec{r} = \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix}.$$

Now compute the following or say why they are undefined:

(a) $A - B, A + B, B - D, \vec{r} - \vec{w}$
(b) $A^T, C^T, D^T, \vec{v}^T, \vec{r}^T$
(c) $2B, 7\vec{w}, 2\vec{u} + 3\vec{w}, -D$
(d) $\vec{v} \bullet \vec{w}, |\vec{v}|, |\vec{w}|, |\vec{u}|, |\vec{r}|, \vec{r} \bullet \vec{u}, \vec{v} \bullet \vec{u}$
(e) Find the angles between the vectors $\vec{v}$ and $\vec{w}$ and the vectors $\vec{r}$ and $\vec{u}$.
(f) $\vec{v}\vec{w}, \vec{w}\vec{v}, \vec{v}\vec{u}, \vec{r}\vec{u}$ (viewing the vectors as matrices)
(g) $A\vec{r}, C\vec{v}, D\vec{r}, B\vec{r}$
(h) CD, DC, AB

4. True False It is possible to find an inverse to a 2×3 sized matrix.

5. True False The determinant of a matrix A will determine whether the system $A\vec{x} = \vec{b}$ will have a unique solution or not, but cannot distinguish between systems with no solutions and infinitely many solutions.

6. True False The system $A\vec{x} = \vec{0}$ will never have no solutions, even if $\det(A) = 0$.

7. Let

$$A = \begin{pmatrix} -1 & 2 & 0 \\ 3 & 0 & 1 \\ 2 & 1 & -1 \end{pmatrix}, B = \begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 1 \\ -1 & -2 & 1 \end{pmatrix}, C = \begin{pmatrix} 0 & 2 & 3 \\ -1 & 1 & 5 \\ 3 & -1 & -12 \end{pmatrix}$$

$$D = \begin{pmatrix} 2 & -3 \\ 3 & -4 \end{pmatrix}, E = \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}, F = \begin{pmatrix} 2 & -3 \\ 6 & -9 \end{pmatrix}, \vec{b}_1 = \begin{pmatrix} 4 \\ -4 \end{pmatrix}, \vec{b}_2 = \begin{pmatrix} 4 \\ 2 \\ -7 \end{pmatrix}$$

(a) Calculate the determinants of A, B, C, D, E, F.

(b) Calculate D^{-1}, E^{-1}, F^{-1}

(c) Write the systems of linear equations that correspond to $E\vec{x} = \vec{b}_1$.

(d) Which of the following systems of equations will have a unique solution?

$$A\vec{x} = \vec{b}_2, \quad B\vec{x} = \vec{b}_2, \quad C\vec{x} = \vec{b}_2, \quad D\vec{x} = \vec{b}_1, \quad E\vec{x} = \vec{b}_1, \quad F\vec{x} = \vec{b}_1$$

8. Compute the following determinants below:

$$\begin{vmatrix} 4 & -3 & 2 \\ 1 & -3 & 1 \\ 9 & 1 & 0 \end{vmatrix}, \qquad \begin{vmatrix} 6 & 7 & -2 \\ -2 & -3 & 1 \\ 7 & 7 & 1 \end{vmatrix}$$

9. True False If a row like $(00\cdots0|1)$ appears during Gaussian elimination, then there are no solutions.

10. True False If a row like $(00\cdots0|0)$ appears during Gaussian elimination, then there are infinitely many solutions.

11. True False If the augmented matrix $(A|\vec{b})$ is reduced into $(I|\vec{c})$ for some vector $\vec{c}$ by Gaussian elimination, then $A\vec{c} = \vec{b}$.

12. Use Gaussian elimination to reduce the following augmented matrices:

(a) $\left(\begin{array}{ccc|c} 2 & 1 & 8 & 1 \\ -1 & 1 & -1 & 0 \\ -2 & 5 & 4 & 1 \end{array}\right)$

(b) $\left(\begin{array}{ccc|c} 4 & 5 & 0 & 2 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 3 & 0 \end{array}\right)$

(c) $\left(\begin{array}{ccc|c} 3 & 2 & 5 & 2 \\ 2 & 0 & 1 & 5 \\ 0 & 0 & 4 & -4 \end{array}\right)$

13. Use Gaussian elimination to find the solution to the following system of equations:

$$\begin{cases} 2x + 3y - z = 0 \\ x + 2y + z = 3 \\ x + 3y + 3z = 7 \end{cases}$$

14. Use Gaussian elimination to find the inverse of the matrix $\begin{pmatrix} 1 & 2 & -1 \\ -3 & 1 & 2 \\ -2 & 2 & 1 \end{pmatrix}$.