Math 10A
Homework #10; Due Friday, 7/27/2018
Instructor: Roy Zhao

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1. True False Changing the initial conditions for a linear homogeneous recurrence relation does not affect the bases of the exponential functions that appear in the formula for the solution.

2. True False Checking that $a_n = f(n)$ is a solution to a recurrence relation may not possible if we do not know how to find the general solution of the recurrence relation.

3. Solve $a_n = a_{n-1} + 2a_{n-2}$ with $a_0 = 2, a_1 = 1$.

4. Find the general solution to $a_n = 2a_{n-1} + 3a_{n-2}$.

5. Solve $a_n = -6a_{n-1} - 9a_{n-2}$ with $a_0 = 3, a_1 = -3$.

6. Solve $a_n = 4a_{n-2}$ with $a_0 = 0, a_1 = 4$.

7. Find a second order linear homogeneous recurrence relation such that $a_n = 3^n - 2^n$ is a solution to it.

8. Find a second order linear homogeneous recurrence relation such that $a_n = n - 2$ is a solution to it. (Hint: $1 = 1^n$)

9. True False The function $f(x) = \dfrac{\ln(x)}{x}$ is a solution to the DE $x^2y' + xy = 1$.

10. Verify that $y = te^t + 1$ is a solution to $y'' - 2y' = 1 - y$.

11. Verify that $y = 2e^{1/(2t)}$ is a solution to $2t^2y' + y = 0$.

12. For the following differential equations, find their order and determine whether they are homogeneous, linear, and/or have constant coefficients.

    (a) $y'' = 2y$

    (b) $y' = y^2$

    (c) $y' + (e^t - \sin(t))y = \tan(t)$

    (d) $y' - ty = t^2$

13. True False When solving a second order linear homogeneous DE with constant coefficients, if both roots of the characteristic polynomial are equal to r, the general solution is of the form $(at + b)e^{rt}$

14. True False All IVPs for second order linear homogeneous DEs with constant coefficients have only one solution.

15. True False All BVPs for second order linear homogeneous DEs with constant coefficients have either no solutions, only one solution, or infinitely many solutions.

16. Find the general solution to the following DEs:

    (a) $y'' + 2y' - 3y = 0$

    (b) $y'' + 5y' = 0$

    (c) $y'' + 2y' + 5y = 0$

17. Solve the IVP $y'' - 3y' + 2y = 0$ with $y(0) = 0, y'(0) = 2$.

18. Solve the BVP $y'' = -4y$ with $y(0) = 0$, $y(\pi) = 1$.

19. Find a second order linear DE whose general solution is $y = c_1 e^{2t} + c_2 e^{-3t}$.

20. Find a second order linear DE such that $y = e^{3t}\sin(t)$ is a solution to it.

21. True False Before trying to find the integrating factor, we must make sure that the coefficient of y' is 1.

22. True False An ODE is both linear and separable exactly when it is of the form $y' = (ay + b)f(t)$ for some function $f(t)$.

23. True False There may be missing solutions when using separable equations because we divide by a function of y.

24. Solve the following DEs and IVPs:

    (a) $y' = t + 5y$

    (b) $y' + y = \sin(e^x)$

    (c) $y' - 2ty = 3t^2 e^{t^2}$, $y(0) = 5$.

    (d) $xy' = y + x^2\sin(x)$, $y(\pi) = 0$.

25. State whether the following functions are separable:

    (a) $\dfrac{1}{yt}$

    (b) $\sin(y)$

    (c) $t\ln y + t$

    (d) $te^y + t^2$

26. Solve the IVP $y' = \dfrac{1 + 3t^2}{3y^2 - 6y}$ with $y(0) = 1$. You do not need to solve explicitly for $y =$.

27. Solve the following ODEs:

    (a) $y' = \dfrac{3t^2}{2y}$

(b) $y' + y^2 \sin(t) = 0$

(c) $ty' = 1 + y^2$

(d) $y' = (y^2 - y)\sin(t)$