Math 10A
Final; Thursday, 8/9/2018
Time: 2:10 PM **Name:** ____________________
Instructor: Roy Zhao **Student ID:** ____________________

- **DO NOT OPEN THE FINAL UNTIL TOLD TO DO SO!**
- Do all problems as best as you can. The exam is 110 minutes long. You may not leave during the last 30 minutes of the exam.
- Use the provided sheets to write your solutions. You may use the back of each page for the remainder of your solutions; in such a case, put an arrow at the bottom of the page and indicate that the solution continues on the back page. **No extra sheets of paper can be submitted with this exam!**
- The exam is closed notes and book, which means: **no class notes, no review notes, no textbooks, and no other materials can be used during the exam.** You can only use your cheat sheet. The cheat sheet is two sides of one regular 8×11 sheet, handwritten.
- **NO CALCULATORS ARE ALLOWED DURING THE EXAM!**
- Justify all your answers, include all intermediate steps and calculations, and box your answers.

1. (10 points) Find the following limits.

   (a) (2 points) $\lim_{x \to 1} \dfrac{x^2 + 1}{x + 2} =$

   (b) (3 points) $\lim_{x \to 0} \dfrac{e^x - 1 - x}{x^2} =$

   (c) (5 points) $\lim_{x \to \infty} \sqrt{x^2 + x} - x =$

2. (15 points) For each part, find $\dfrac{dy}{dx}$.

(a) (5 points) $y = e^{\sin(x^2)}$.

(b) (5 points) $x^2 + y^2 = xy$. (You can leave your answer in terms of x and y)

(c) (5 points) $y = \displaystyle\int_1^{\sqrt{x}} \frac{t^2}{1+t^2}\,dt$.

3. (10 points) Find the following integrals.

   (a) (5 points) $\int 2x^3\sqrt{x^2-1}dx =$

   (b) (5 points) $\int 4x^3 \arctan(x^2)dx =$

4. (5 points) Does $\displaystyle\int_2^\infty \frac{1}{\sqrt{x^2-1}}dx$ converge?

5. (10 points) Suppose that I am currently standing 5 meters east of a bus. The bus starts moving north at a rate of 6 m/s. How fast is the bus moving away from me two seconds later?

6. (10 points) Find a second order recurrence relation or differential equation that has the following solutions.

   (a) (5 points) $a_n = 2^n - 1$.

   (b) (5 points) $y = e^{2t}\sin(t)$.

7. (10 points) Solve the following IVPs.

   (a) (5 points) $xy' = 2y + 2x^4, y(1) = 2.$

   (b) (5 points) $e^{t^2}y' = ty^2, y(0) = 1.$

8. (10 points) Let $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 5 & 1 \\ -4 & -8 & 7 \end{pmatrix}$ and $\vec{v} = \begin{pmatrix} -2 \\ 2 \\ 3 \end{pmatrix}$.

(a) (2 points) Find $A\vec{v}$.

(b) (8 points) Use Gaussian elimination to find the solution to $A\vec{x} = \vec{v}$.

9. (10 points) Find the general solution to the system of differential equations

$$\begin{cases} y_1'(t) = 3y_2(t) \\ y_2'(t) = y_1(t) - 2y_2(t) \end{cases}.$$

10. (10 points) Bubble True or False. (1 point for correct answer, 0 if incorrect)

(a) Ⓣ Ⓕ If $f(x)$ is not defined at $x = 2$, then $\lim_{x \to 2} f(x)$ doesn't exist.

(b) Ⓣ Ⓕ The graph of $f(x-1)$ is the graph of $f(x)$ shifted 1 unit to the left.

(c) Ⓣ Ⓕ Using Simpson's method will give the exact answer when integrating $\int_0^1 x^3 + 3x^2 + 1dx$ with $n = 2$.

(d) Ⓣ Ⓕ Changing the initial conditions for a linear homogeneous recurrence relation does not affect the bases of the exponential functions that appear in the formula for the solution.

(e) Ⓣ Ⓕ BVPs for second order linear homogeneous DEs with constant coefficients have either no solutions or infinitely many solutions.

(f) Ⓣ Ⓕ The slope field of $\frac{dy}{dt} = \sin(t)$ will be the same if we shift it up or down.

(g) Ⓣ Ⓕ If we find two distinct solutions to $A\vec{x} = \vec{b}$, then $|A| = 0$.

(h) Ⓣ Ⓕ If the augmented matrix $(A|\vec{b})$ is reduced into $(I|\vec{c})$ for some vector $\vec{c}$ by Gaussian elimination, then $A\vec{c} = \vec{b}$.

(i) Ⓣ Ⓕ An eigenvector can be the zero vector.

(j) Ⓣ Ⓕ If 2 is an eigenvalue for A, then 4 is an eigenvalue for A^2.