

MATH 128A Numerical Analysis Discussion Section

Final Proejct

- <https://bcourses.berkeley.edu/courses/1542890/files/folder/Lecture%20Notes?preview=91266281>

The screenshot displays the Berkeley Courseware interface for the course MATH 128A-LEC-001. The left sidebar shows a navigation menu with options like Home, Announcements, Assignments, Discussions, Grades, People, Pages, Files (selected), Syllabus, Ed Discussion, Outcomes, Rubrics, and Quizzes. The main content area shows the 'Lecture Notes' folder. A search bar and a toolbar with icons for viewing, downloading, and other actions are visible. A list of files is shown, with the first file, 'MA128AFinalProject_Spring_2025_.pdf', highlighted by a red circle. The other files in the list are 'MA128ALecture_midterm_Review__Spring_2025_.pdf', 'MA128ALectureWeek1__Spring_2025_.pdf', and 'MA128ALectureWeek2__Spring_2025_.pdf'.

Brief Review

- Higher-Order ODEs
 - Equivalent linear system of 1st order ODE
- Multistep method
 - Adams-Bashforth
 - Adams-Moulten
- Stability
 - Consistent + Stable = Convergent
 - Strongly/Weakly/Unstable
- Stiffness
 - A-stable

Higher-order ODEs

- m-th order system

$$u_i' = f_i(t, u_1, \dots, u_m)$$

- Higher order

- Take $u_i = u_{i-1}'$

- Higher order Lip. Conti.

$$|f(t, u) - f(t, v)| \leq L \|u - v\|_1 \quad u, v \in R^n$$

Multi-step method

- Numerical scheme using multiple $f(t_k, y(t_k))$

- (Forward) Euler

$$y_{k+1} = y_k + hf(t_k, y_k)$$

- Trapezoidal

$$y_{k+1} = y_k + \frac{h}{2} (f(t_k, y_k) + f(t_{k+1}, y_{k+1}))$$

- Adams-Bashforth
- Adams-Moulton

Multi-step method

- Basic idea of the construction

- Approximate

$$\int_{t_n}^{t_{n+1}} f(t, y(t)) dt$$

- Using the interpolation -> LMM

- Excluding the right endpoint -> explicit (open)
 - Including the right endpoint -> implicit (close)

- Using the quadrature rule -> RK mthd / GL mthd

Stability

- Consistent
 - $LTE \rightarrow 0$ as $h \rightarrow 0$
- Stable
 - Resistance against small perturbation
 - I.e. perturbation remain bounded in the limit $h \rightarrow 0$
- Convergent
 - Global error $\rightarrow 0$ as $h \rightarrow 0$
 - Consistent + Stable

Stability

- Stability test

- Check root condition -> the roots of characteristic polynomial($P(\lambda)$)

$$P(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - \dots - a_1\lambda - a_0.$$

- Strongly stable

$$|\lambda| \leq 1 \& |\lambda| = 1 \text{ iff } \lambda = 1$$

- Weakly stable

$$|\lambda| \leq 1 \& |\lambda| = 1 \rightarrow \lambda \text{ is simple}$$

- Unstable

O. W.

Stiffness

- The region R of absolute stability
 - New characteristic polynomial $Q(z, h\lambda)$

$$w_{i+1} = Q(h\lambda)w_i$$

$$w_{j+1} = a_{m-1}w_j + \cdots + a_0w_{j+1-m} + h\lambda(b_mw_{j+1} + b_{m-1}w_j + \cdots + b_0w_{j+1-m}),$$

$$Q(z, h\lambda) = (1 - h\lambda b_m)z^m - (a_{m-1} + h\lambda b_{m-1})z^{m-1} - \cdots - (a_0 + h\lambda b_0)$$

- R

$$R = \{h\lambda \in \mathcal{C} \mid |Q(h\lambda)| < 1\}.$$

$$\tilde{R} = \{h\lambda \in \mathcal{C} \mid |\tilde{\beta}_k| < 1, \text{ for all zeros } \beta_k \text{ of } Q(z, h\lambda)\}.$$