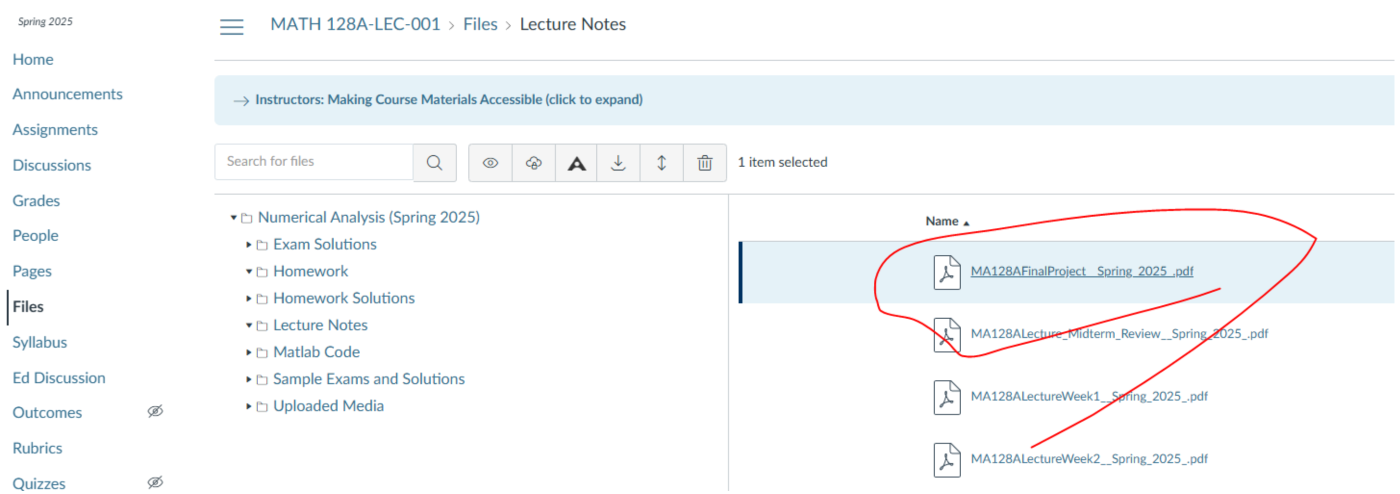
MATH 128A Numerical Analysis Discussion Section

Final Proejct

- https://bcourses.berkeley.edu/courses/1542890/files/folder/Lecture%20Notes?preview=91266281



Brief Review

- Higher-Order ODEs
  - Equivalent linear system of 1^{st} order ODE
- Multistep method
  - Adams-Bashforth
  - Adams-Moulten
- Stability
  - Consistent + Stable = Convergent
  - Strongly/Weakly/Unstable
- Stiffness
  - A-stable

Higher-order ODEs

- m-th order system

$$u_i' = f_i(t, u_1, \dots, u_m)$$

- Higher order
  - Take $u_i = u_{i-1}'$
- Higher order Lip. Conti.

$$|f(t,u) - f(t,v)| \leq L\|u - v\|_1 \; u, v \in R^n$$

Multi-step method

- Numerical scheme using multiple $f\left(t_k, y(t_k)\right)$
  - (Forward) Euler
$$y_{k+1} = y_k + hf(t_k, y_k)$$
  - Trapezoidal
$$y_{k+1} = y_k + \frac{h}{2}\left(f(t_k, y_k) + f(t_{k+1}, y_{k+1})\right)$$
  - Adams-Bashforth
  - Adams-Moulton

Multi-step method

- Basic idea of the construction
  - Approximate

$$\int_{t_n}^{t_{n+1}} f\big(t, y(t)\big)dt$$

  - Using the interpolation -> LMM
    - Excluding the right endpoint -> explicit (open)
    - Including the right endpoint -> implicit (close)
  - Using the quadrature rule -> RK mthd / GL mthd

Stability

- Consistent
  - LTE -> 0 as h -> 0
- Stable
  - Resistance against small perturbation
  - I.e. perturbation remain bounded in the limit h -> 0
- Convergent
  - Global error -> 0 as h -> 0
  - Consistent + Stable

Stability

- Stability test
  - Check root condition -> the roots of characteristic polynomial(P(\lambda))

$$P(\lambda) = \lambda^m - a_{m-1}\lambda^{m-1} - a_{m-2}\lambda^{m-2} - \cdots - a_1\lambda - a_0.$$

  - Strongly stable

$$|\lambda| \le 1 \& |\lambda| = 1 \; iff \; \lambda = 1$$

  - Weakly stable

$$|\lambda| \le 1 \& |\lambda| = 1 \; \rightarrow \lambda \; is \; simple$$

  - Unstable

$$o.w.$$

Stiffness

- The region R of absolute stability
  - New characteristic polynomial Q(z, h\lambda)

$$w_{i+1} = Q(h\lambda) w_i$$

$$w_{j+1} = a_{m-1} w_j + \cdots + a_0 w_{j+1-m} + h\lambda(b_m w_{j+1} + b_{m-1} w_j + \cdots + b_0 w_{j+1-m}),$$

$$Q(z, h\lambda) = (1 - h\lambda b_m) z^m - (a_{m-1} + h\lambda b_{m-1}) z^{m-1} - \cdots - (a_0 + h\lambda b_0)$$

  - R

$$R = \{h\lambda \in \mathcal{C} \mid |Q(h\lambda)| < 1\}$$

$$R = \{h\lambda \in \mathcal{C} \mid |\beta_k| < 1, \text{ for all zeros } \beta_k \text{ of } Q(z, h\lambda)\}.$$