MATH 128A Numerical Analysis Discussion Section

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Brief Review

- Hermite Interpolation
  - Divided difference with f'(x)
- Cubic Spline Interpolation
  - Boundary conditions
  - Better than the previous interpolation?

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Hermite interpolation

x	$f(x)$	First divided differences	Second divided differences	Third divided differences
x_0	$f[x_0]$			
		$f[x_0, x_1] = \dfrac{f[x_1] - f[x_0]}{x_1 - x_0}$		
x_1	$f[x_1]$		$f[x_0, x_1, x_2] = \dfrac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$	
		$f[x_1, x_2] = \dfrac{f[x_2] - f[x_1]}{x_2 - x_1}$		$f[x_0, x_1, x_2, x_3] = \dfrac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0}$
x_2	$f[x_2]$		$f[x_1, x_2, x_3] = \dfrac{f[x_2, x_3] - f[x_1, x_2]}{x_3 - x_1}$	
		$f[x_2, x_3] =$ f'(x2)		$f[x_1, x_2, x_3, x_4] = \dfrac{f[x_2, x_3, x_4] - f[x_1, x_2, x_3]}{x_4 - x_1}$
x_3	$f[x_3]$		$f[x_2, x_3, x_4] = \dfrac{f[x_3, x_4] - f[x_2, x_3]}{x_4 - x_2}$	
		$f[x_3, x_4] = \dfrac{f[x_4] - f[x_3]}{x_4 - x_3}$		$f[x_2, x_3, x_4, x_5] = \dfrac{f[x_3, x_4, x_5] - f[x_2, x_3, x_4]}{x_5 - x_2}$
x_4	$f[x_4]$		$f[x_3, x_4, x_5] = \dfrac{f[x_4, x_5] - f[x_3, x_4]}{x_5 - x_3}$	
		$f[x_4, x_5] = \dfrac{f[x_5] - f[x_4]}{x_5 - x_4}$		
x_5	$f[x_5]$			

Hermite interpolation

Table 3.17

1.3	0.6200860					
		−0.5220232				
1.3	0.6200860		−0.0897427			
		−0.5489460		0.0663657		
1.6	0.4554022		−0.0698330		0.0026663	
		−0.5698959		0.0679655		−0.0027738
1.6	0.4554022		−0.0290537		0.0010020	
		−0.5786120		0.0685667		
1.9	0.2818186		−0.0084837			
		−0.5811571				
1.9	0.2818186					

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Cubic spline interpolation

- Find piece-wise cubic polynomial function S(x)
  - $S(x) \in C^2(x_0, x_n)$
  - $S(x_i) = f(x_i)$
  - Natural / Clamped condition
- Above conditions are equivalent to
  - $S_j(x_j) = f(x_j), S_{j+1}(x_{j+1}) = f(x_{j+1})$
  - $S_j(x_j) = S_{j+1}(x_j) = f(x_j)$
  - $S_j'(x_j) = S_{j+1}'(x_j)$
  - $S_j''(x_j) = S_{j+1}''(x_j)$

Cubic spline interpolation

- Natural
  - $S_0''(x_0) = S_{n-1}''(x_n) = 0$
- Clamped
  - $S_0'(x_0) = f'(x_0)$
  - $S_{n-1}'(x_n) = f'(x_n)$

Cubic spline interpolation

- Better than the previous interpolation?
  - Comparing the error terms

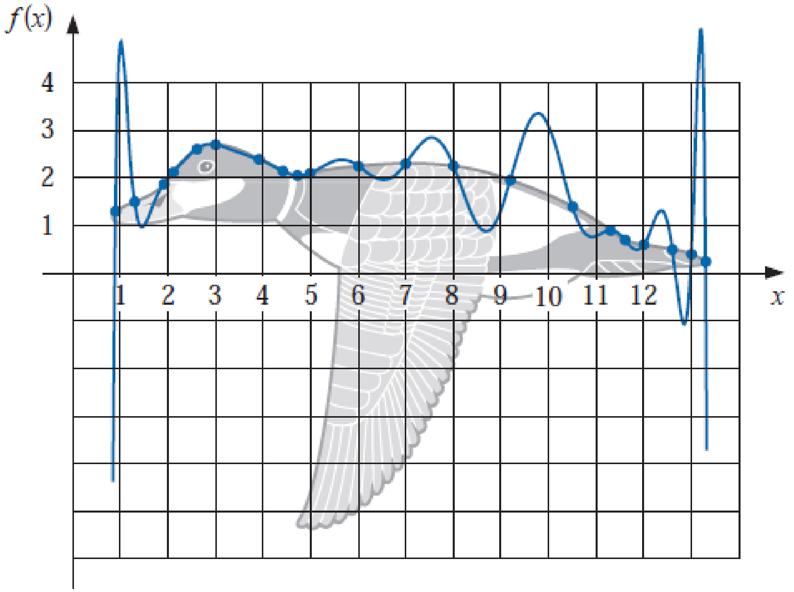
$$\frac{f^{(n+1)}\left(\xi(x)\right)}{(n+1)!}(x-x_0)\cdots(x-x_n)$$

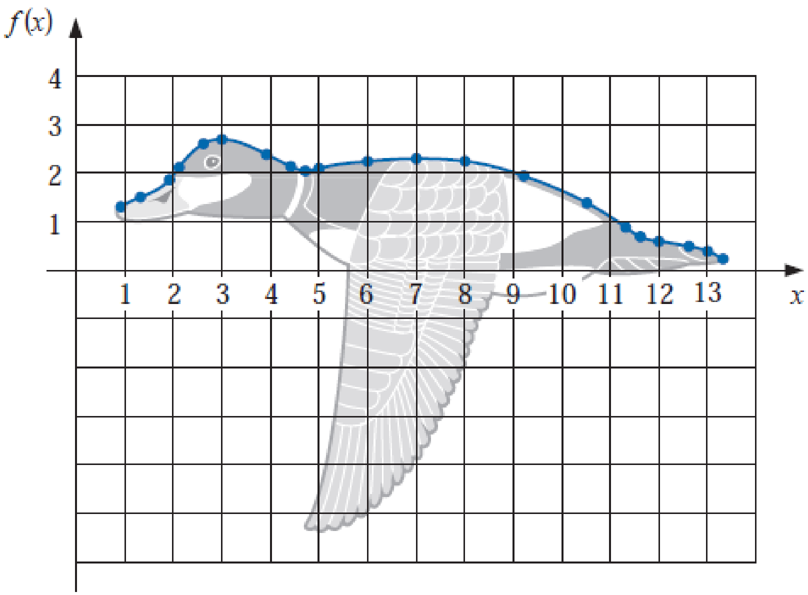
vs

$$\frac{5\max\left(f^{(4)}(x)\right)}{384}\left(x_{j+1}-x_j\right)^4$$

Cubic spline interpolation

- Better than the previous interpolation?





Programming Exercises

- Goal
  - Derive the spline function from the given dataset
- Example

1. Determine the natural cubic spline S that interpolates the data $f(0) = 0$, $f(1) = 1$, and $f(2) = 2$.

- Exercise

2. Determine the clamped cubic spline s that interpolates the data $f(0) = 0$, $f(1) = 1$ and $f(2) = 2$ and satisfies $s'(0) = s'(2) = 1$.