MATH 128A Numerical Analysis Discussion Section

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Brief Review

- Order of Convergence
  - Modified Newton's method
- Accelerating Convergence
  - Aitken's Δ^2 Method
  - Steffensen's Method
- Lagrange Interpolation Polynomial
  - $p_{\mathrm{n},i}(x_j) = \delta_{i,j}$

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Newton's method

- f(x) is continuous and differentiable

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

- Converging very fast.

- Q. What if $f'(p) = 0$?
  A. Can not guarantee fast convergence.
  Indeed, it merely linearly converges.

Modified Newton's method

- f(x) is continuous and differentiable
- multiplicity m of root x
  - Simply $f(a) = f'(a) = \cdots = f^{(m-1)}(a) = 0$
- $x_{n+1} = x_n - \frac{f(x_n)f'(x_n)}{f'(x_n)^2 - f(x_n)\boldsymbol{f''(x_n)}}$
- Quadratically convergent but impractical

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  - Another measurement
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Aitken's Δ^2 Method

$$\widehat{p_n} = p_n - \frac{(p_{n+1} - p_n)^2}{p_{n+2} - 2p_{n+1} + p_n}$$

- Accelerate the convergence of a sequence that is linearly convergent.
- How to use?

  Calculate p_n until the error is $\sqrt{Tol}$ and apply Aitken's Δ^2 Method

Steffensen's Method

$$p_0^{(i)} = \{\Delta^2\}\left(p_0^{(i-1)}\right)$$
$$p_1^{(i)} = g\left(p_0^{(i)}\right)$$
$$p_2^{(i)} = g\left(p_1^{(i)}\right)$$
$$p_0^{(i+1)} = \{\Delta^2\}\left(\boldsymbol{p_0^{(i)}}\right)$$

- Accelerate the order of convergence of the given fixed point method using Aitken's method

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Lagrange polynomial

- Polynomials s.t. $p_{\mathrm{n},i}(x_j) = \delta_{i,j}$ for given $\{x_j\}_{j=0,\ldots,n}$

$$p_{\mathrm{n},i}(x) = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

- Interpolation result

$$Q(x) = \sum f(x_i) p_{n,i}(x)$$

Programming Exercises

- Goal : Use code in the web
- http://persson.berkeley.edu/math128a/
- Example problem

4. Let $g(x) = 1 + (\sin x)^2$ and $p_0^{(0)} = 1$. Use Steffensen's method to find $p_0^{(1)}$ and $p_0^{(2)}$.

- Exercise
  - Solve following problem using steffensen_table.m

7. Use Steffensen's method to find, to an accuracy of 10^{-4}, the root of $x^3 - x - 1 = 0$ that lies in $[1, 2]$ and compare this to the results of Exercise 8 of Section 2.2.