MATH 128A Numerical Analysis Discussion Section

Raehyun Kim*

* Department of Mathematics, UC Berkeley

Announcement

- First Quiz : Feb. 7 in DIS section. Covers Sec 1.1, 1.2, 1.3, 2.1 (HW #1).
- Each quiz will cover contents from two previous homeworks

Announcement

- Account Information
  - Username : !cmfmath128a
  - Password : c@1b3arsdivide

Brief Review

- Finite-digit arithmetic
  - Order does matter
  - Addition (rounding)
    - (10 + 3.4) + 3.4 = 13 + 3.4 = 16
    - 10 + (3.4 + 3.4) = 10 + 6.8 = 17
  - Multiplication (chopping)
    - (2.5 X 3.7) X 4.8 = 9.2 X 4.8 = 44
    - 2.5 X (3.7 X 4.8) = 2.5 X 17 = 42

Brief Review

- Error analysis
  - Error formula
    - Absolute error $|p - p^*|$
    - Relative error $\frac{|p-p^*|}{|p|}$
  - abs/rel error
    - Let approximate 2^{10} to 1000
    - Abs error : 24
    - Rel error : 24/1024 = 0.0234375 = 2.34375e-2

Brief Review

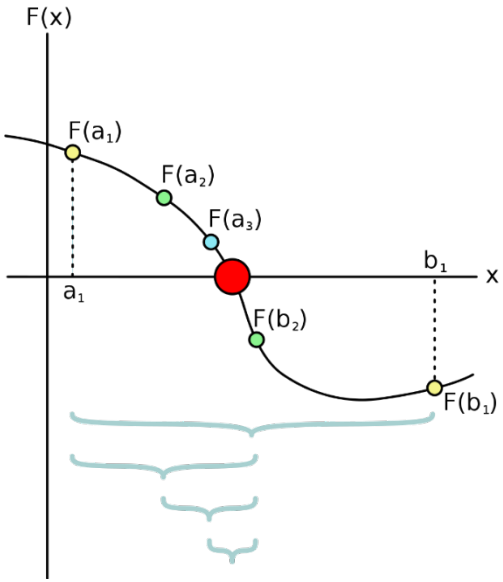
- Rate of convergence
  - $\{\alpha_n\}_{n\in\mathbb{N}}$, $|\alpha_n - \alpha| \leq K|\beta_n|$. Then this seq. converges with rate of convergence $\mathrm{O}(\beta_n)$.
  - Basically, the rate of convergence measures the convergence speed of a (well-known) seq.
  - Ex $\lim_{h\to 0}(1-\cos h) = 0$. How fast?

Brief Review

- $\lim_{h \to 0} |1 - \cos h| = 0$
  - convergence is faster than $O(h^0)$
- $\lim_{h \to 0} \frac{|1-\cos h|}{|h|} = 0$
  - convergence is faster than $O(h^1)$
- $\lim_{h \to 0} \frac{|1-\cos h|}{|h^2|} = \frac{1}{2}$
  - Finally we have the rate of convergence $O(h^2)$

Brief Review

- Bisection method
  - Based on the Intermediate value theorem
  - half -> half -> …
  - Very slow, but reliable



Programming Exercises

- Goal
  - Define function
  - Use symbolic library
- Core
  - f = @(t) t*exp(t^2)
  - syms x
  - taylor
  - matlabFunction
- Example case

15. Find the fourth Taylor polynomial $P_4(x)$ for the function $f(x) = xe^{x^2}$ about $x_0 = 0$.

a. Find an upper bound for $|f(x) - P_4(x)|$, for $0 \le x \le 0.4$.

Programming Exercises

- Goal
  - Define function
  - Use symbolic library
- Exercise
  - $\mathrm{f(x)} = 2x\cos 2x - (x-2)^2, x_0 = 0$
    - Define function $f(x)$
    - Compute $f(0.4)$
    - Compute 4th order Tayler polynomial $P_4(x)$
    - Compute the error $|f(0.4) - P_4(0.4)|$